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"Men go grey": Robert Kilwardby and the Logic of Natural Contingency

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Logical compendia and commentaries on Aristotle's *Prior Analytics* in the twelfth and thirteenth centuries divide the senses of "possible" in a standard way. First, possibility is divided into a broad sense in which only the impossible is excluded (but which is compatible with necessity), and a narrower sense that excludes both necessity and impossibility (i.e., a sense of "possible" meaning "neither necessary nor impossible").[1] Although medieval logicians did not use any notation resembling that of contemporary modal logic, it is not hard to see how to express this distinction using contemporary notation: If we use $\Diamond$ to represent possibility in the broad sense, then a statement p is possible in the narrow sense if and only if $\Diamond p \wedge \Diamond \neg p$. I will refer to the narrower sense of possibility as "contingency".[2]

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[1] See 'Anonymus Aurelianensis III' in Aristotelis *Analytica priora* [36, pp. 98.3–6, 99.27–28], Lambert of Auxerre [1, pp. 39–41], Robert Kilwardby [35, pt. 1, p. 376.235–243], Albert the Great [4, tract. I, c. 12]. Some authors in this period also, confusingly, delineate a third sense of possibility that they identify with necessity. This may be the result of confusing the claims that some possibilities in the broad sense are necessary (which is true) with the claim that "necessary" is one meaning of "possible" (which is false): On this see Thom [31, pp. 29–30] and Lagerlund [16, p. 23]. I will set aside this putative sense of possibility in what follows.

[2] Modern scholars of medieval logic sometimes also call this "two-sided" [31, p. 138] or "two-edged" [14, p. 531] possibility. I follow the usage of Thom [34] in calling the narrow sense "contingent". Most logicians of the thirteenth century do not

Contingency is then further divided into at least two sorts: "Indeterminate" contingency (*contingens infinitum*) and "natural" contingency (*contingens natum*).[3] This distinction tends to be glossed in terms of how the contingency is related towards being and non-being. "Indeterminate" contingencies, which are illustrated by chance events, are said to be related "equally" towards being and non-being,[4] whereas a "natural" contingency is said to be related "more" towards being,[5] or "unequally" towards being and non-being.[6] The stock example of the latter type of contingency was "men go grey in old age", or sometimes just "men go grey".[7]

Unlike the distinction between broad possibility and contingency, the distinction between natural and indeterminate contingency does not readily suggest any translation into contemporary modal logic. Indeed, the sense of the distinction is not really made clear by this standard gloss.

distinguish the terms *possibile* and *contingens* in this way, since these were Boethius's translations of Aristotle's terms δύνατον and ἐνδεχόμενον respectively, and neither Aristotle nor Boethius employed these terms to mark the distinction between broad and narrow possibility. The sub-types of contingency are, however, usually referred to as *contingens natum* and *contingens infinitum*. On the terminological issues see Knuuttila [15, pp. 106–7], Lagerlund [16, p. 23] and Hintikka [11, pp. 27–40]. I will follow the modern usage and always use "contingent" and "contingency" to refer to narrow possibility, except in quotations.

[3] See 'Anonymus Aurelianensis III' in Aristotelis *Analytica priora* [36, p. 100.2–3], *Dialectica Monacensis* [7, p. 481.14–21], Lambert of Auxerre [1, p. 42.14–26], Robert Kilwardby [35, pt. 1, pp. 368.123–370.127], Albert the Great [4, tract. I, c. 12–15]. *Contingens infinitum* is also sometimes referred to as *contingens ad utrumlibet*, but the latter expression is used in some early texts to refer to possibility in the broad sense rather than indeterminate contingency; on this see Knuuttila [14, p. 532, 15, p. 112], Jacobi [12, pp. 92–4] and Lagerlund [16, pp. 24–25].

[4] *aequaliter se habet ad esse et ad non esse*: Albert the Great [4, tract. I, c. 12], Robert Kilwardby [35, pt. 1, p. 370.136–7]. *non magis se habet ad esse quam ad non esse*: 'Anonymus Aurelianensis III' in Aristotelis *Analytica priora* [36, p. 100.30–31], Lambert of Auxerre, [1, p. 42.21–22]. *sive 'infinitum', sive 'aequale'*: Roger Bacon [6, 2.1, §392]. On choice and chance, see note 11.

[5] *magis se habet ad esse quam ad non esse* [1, p. 42.17].

[6] *non equaliter se habet ad esse et non esse* [35, pt. 1, p. 398.581].

[7] *Hominem canescere in senectute*: Roger Bacon [6, 2.1, §392], 'Anonymus Aurelianensis III' in Aristotelis *Analytica priora* [36, p. 100.10], Lambert of Auxerre [1, p. 42.16]. *Hominem canescere*: Robert Kilwardby [35, pt. 1, p. 370.131] (but see [35, pt. 1, p. 394.530–542], where Kilwardby seems aware of the usual addition *in senectute*).

In what way does the natural tendency of men to develop grey hair, or to do so in old age, make this a predication which is more "related towards being" than a chance event? What, for that matter, does it mean for a contingent predication to be related "more towards being", and in what way does this render a contingency "natural"?

Robert Kilwardby (c. 1215–1279), a Parisian Master of Arts and later Archbishop of Canterbury, is among the first Latin commentators to go beyond the standard tropes and give an account of why certain contingencies are distinctively natural. He does so in his extensive question-commentary on Aristotle's *Prior Analytics*,[8] in which he also covers the logic of natural contingency, treating it as a modality within a system of syllogistic modal logic on the basis of the meaning he takes the expression *contingens natum* to have.

My goal in what follows is, first, to explain how Kilwardby develops his notion of natural contingency in the course of addressing exegetical problems that arise in interpreting *Prior Analytics* I.13. I then show how Kilwardby's way of understanding the distinction allows him to sketch a logic of natural processes and their results that may be formalized in a temporal free logic. Since Kilwardby gives this account in the course of his commentary on Aristotle, it is important to first review the passage that forms the basis for Kilwardby's understanding of the distinction between kinds of contingency and the exegetical problems that this passage gives rise to.

1 Robert Kilwardby's commentary on *Prior Analytics* I.13

In *Prior Analytics* I.13, Aristotle commences his treatment of syllogisms with premises containing a modality of contingency. Aristotle defines "being possible" and "the possible" as "that which, while not being necessary, will not lead to anything impossible when it is assumed to

[8] The critical edition and translation of Kilwardby's commentary is due to Thom and Scott [35]. For a brief overview of what is known about Kilwardby's life and career, see Silva [27].

belong" [29, I.13, 32a18–20].[9] This definition rules out both necessity and impossibility, and thus is a definition of what I am calling contingency. After discussing the conversion of contingencies, Aristotle then adds the following remark:

> After these explanations, let us add that 'being possible' is said in two ways: in one way of what happens for the most part, when the necessity has gaps, such as that a man turns grey or grows or ages, or generally what belongs by nature. For this has no continuous necessity because a man does not exist forever, but while a man exists, it happens either of necessity or for the most part. In another way 'being possible' is said of what is indeterminate, that is, what is possible both this way and not this way, such as that an animal walks or that an earthquake happens while it walks, or, generally, what comes about by chance, for this is by nature no more this way than the opposite way [29, I.13, 32b4–13].[10]

This passage announces that it will distinguish two further senses of "being possible," here referring to the narrow sense of contingency. One of these is associated with "what belongs by nature" and the other with "what is indeterminate", which includes what "comes about by chance".[11] "Men go grey" is given as an example of the first kind of contingency, a contingency in the sense of a natural occurrence, which following medieval

[9] λέγω δ' ἐνδέχεσθαι καὶ τὸ ἐνδεχόμενον, οὗ μὴ ὄντος ἀναγκαίου, τεθέντος δ' ὑπάρχειν, οὐδὲν ἔσται διὰ τοῦτ' ἀδύνατον. For the Greek of the *Prior Analytics* and *Posterior Analytics* I rely on the edition of W. D. Ross [26].

[10] Διωρισμένων δὲ τούτων πάλιν λέγωμεν ὅτι τὸ ἐνδέχεσθαι κατὰ δύο λέγεται τρόπους, ἕνα μὲν τὸ ὡς ἐπὶ τὸ πολὺ γίνεσθαι καὶ διαλείπειν τὸ ἀναγκαῖον, οἷον τὸ πολιοῦσθαι ἄνθρωπον ἢ τὸ αὐξάνεσθαι ἢ φθίνειν, ἢ ὅλως τὸ πεφυκὸς ὑπάρχειν (τοῦτο γὰρ οὐ συνεχὲς μὲν ἔχει τὸ ἀναγκαῖον διὰ τὸ μὴ ἀεὶ εἶναι ἄνθρωπον, ὄντος μέντοι ἀνθρώπου ἢ ἐξ ἀνάγκης ἢ ὡς ἐπὶ τὸ πολύ ἐστιν), ἄλλον δὲ τὸ ἀόριστον, ὃ καὶ οὕτως καὶ μὴ οὕτως δυνατόν, οἷον τὸ βαδίζειν ζῷον ἢ βαδίζοντος γενέσθαι σεισμόν, ἢ ὅλως τὸ ἀπὸ τύχης γινόμενον· οὐδὲν γὰρ μᾶλλον οὕτως πέφυκεν ἢ ἐναντίως.

[11] Alexander of Aphrodisias takes the latter class of contingencies to include not only chance events but also the results of free choice [38, 162.31–2]. He is followed in the contemporary literature by Smith [28, pp. 126–7]. Medieval authors tend to employ examples of chance occurrences rather than freely chosen occurrences as the contrast with natural occurrences, but Kilwardby does once mention choice (*propositio*) in this connection [35, pt. 1, p. 370.138].

usage I will call a natural contingency.

Aristotle's characterization of natural contingency in this passage gives rise to significant interpretative difficulties. In particular, it is unclear whether Aristotle takes natural contingencies to be contingent in the usual sense that entails being neither necessary nor impossible. On first glance, Aristotle seems to be identifying natural contingency with "what happens for the most part" [29, I.13, 32b5–6].[12] In that case, his point would be that "men go grey" describes more than a chance occurrence because *most* men go grey (and so it is certainly not impossible); nevertheless, developing grey hair is not a universal or inevitable fate for men and so "men go grey" also falls short of necessity. The predication would then be contingent in the usual sense which requires being neither necessary nor impossible.

What complicates this reading is that Aristotle goes on immediately to say that in the type of contingency exemplified by men going grey, "the necessity has gaps" [29, I.13, 32b5].[13] This suggests that Aristotle means to treat the fact of a man going grey as a type of necessity (albeit one with "gaps"),[14] and so, if it is also a "contingency" it cannot be a contingecy in the usual sense that excludes necessity. The following sentence at first seems to confirm this: Aristotle denies that men going grey has "continuous" [29, I.13, 32b8][15] necessity, apparently suggesting that it does have some sort of "non-continuous" necessity. Yet Aristotle then seems to retreat to the claim that going grey happens "either of necessity or for the most part" [29, I.13, 32b10–11].[16]

This is rather confusing, and can give the impression that Aristotle hedges or equivocates regarding the modal status of "men go grey" and the other examples of natural occurrences given here. The unclarity recurs a few lines below, where Aristotle asks whether there can be scientific knowledge and demonstration of contingencies. The question arises naturally given his view in the *Posterior Analytics* that scientific

[12] ὡς ἐπὶ τὸ πολὺ γίνεσθαι.
[13] διαλείπειν τὸ ἀναγκαῖον.
[14] Cf. Striker [30, pp. 157–159], discussed below.
[15] συνεχὲς.
[16] ἢ ἐξ ἀνάγκης ἢ ὡς ἐπὶ τὸ πολύ ἐστιν.

knowledge and demonstration concerns only what holds of necessity and excludes what takes place on account of chance.[17] In *Prior Analytics* I.13, he maintains that, although there is "no knowledge or demonstrative syllogism of indeterminate things",[18]

> there is knowledge of things that happen by nature, and by and large arguments and investigations are concerned with what is possible in this way [29, I.13, 32b20].[19]

Here, again, Aristotle's answer gives the impression of hedging: What does he mean when he says that "by and large" (σχεδὸν) there is scientific argumentation concerned with what is contingent in the sense of a natural contingency? Is there, or isn't there?

Given these interpretive difficulties, it is not surprising that this passage has elicited a range of interpretations. In contemporary scholarship, this passage has been subjected to close reading by Gisela Striker, who takes Aristotle here to be attempting to reduce natural occurrences to "gappy" necessities, that is, necessities which hold only when some further condition S is satisfied. Aristotle's attempt fails, however, on Striker's view, since the true logical form of such a statement is a conditional under a necessity operator (viz., $\Box(S \to p)$), and Aristotle's syllogistic logic does not give him the resources to properly express this. Aristotle thus, on her view, wavers between treating natural occurrence as a form of contingency and as form of necessity, aware of some of the problems involved in each of these approaches but unable to overcome them [30].[20]

Latin commentators in the twelfth and thirteenth centuries, by contrast, seldom entertain the possibility that Aristotle's view is incoherent, and strive to find a consistent interpretation of the text as they receive it.[21] Among the earliest Latin commentators to attempt an exegesis

[17] See *Posterior Analytics* I.4, 73a21–23; I.6 74b5–6; I.30 87b19; I.33 88b30–32 [26].

[18] I.e., of contingencies that are *not* of the class exemplified by "men go grey".

[19] τῶν δὲ πεφυκότων ἔστι, καὶ σχεδὸν οἱ λόγοι καὶ αἱ σκέψεις γίνονται περὶ τῶν οὕτως ἐνδεχομένων

[20] Another way in which Striker's interpretation differs from the that of the medieval commentators discussed here is that they all take Aristotle to be distinguishing senses of the term "possible", whereas on Striker's view, Aristotle may be instead identifying the types of case in which a contingency holds [30, pp. 150–151].

[21] This is not to say that commentators in this period take Aristotle to be infallible.

of this passage are the author of the 'Anonymus Aurelianensis III' and Robert Kilwardby.[22] They read this passage in Boethius's translation. I reproduce the Florence recension of Boethius's translation of 32b4–13 here:[23]

> Determinatis autem his rursum dicimus quoniam 'contingere' duobus modis dicitur, uno quidem quod plerumque fit et deficit necessarium, ut 'canescere hominem' vel 'augeri' vel 'minui', vel omnino 'quod natum est esse' (hoc enim non continuum quidem habet necessarium eo quod non semper est homo, cum autem homo est aut ex necessitate aut ut in pluribus est); alio autem infinitum, quod et sic et non sic possibile, ut 'animal ambulare' vel 'ambulante fieri terrae motum' vel omnino quod a casu fit; nihil enim magis sic natum est vel e contrario.[24]

Boethius translates the phrase διαλείπειν τὸ ἀναγκαῖον, which Striker renders with "the necessity has gaps", as *deficit necessarium* ("lacks necessity" or "falls short of necessity").[25] This has an important effect

Kilwardby, despite his generally reverent attitude towards Aristotle, is occasionally willing to correct him: See Thom [33, p. 256].

[22] On the dating of the former see note 27 below. On the dating of the latter, see Lagerlund [16, pp. 19–21], according to which Kilwardby's commentary predates Albert the Great's, and probably also Roger Bacon's and Lambert of Auxerre's. The texts edited by de Rijk [7] contain discussion of natural contingency but no systematic interpretation of *Prior Analytics* I.13.

[23] This is probably not, word for word, the text that these commentators read. According to Thom and Scott [35, pp. lxxiv–lxxvi], the collated manuscripts of Kilwardby's text also contain readings that agree with the Chartres recension, and sometimes they deviate from Boethius's translation altogether. Thörnqvist argues that the author of the 'Anonymus Aurelianensis III' worked from a conflated version, containing readings from the Chartres and the Florentine editions of Boethius [37, p. 29]. At least as concerns this passage, I have not found any evidence that these commentators were reading a text that differed from the text that I reproduce here in ways that would affect its sense. Here I am primarily interested in the way the text they read renders διαλείπειν τὸ ἀναγκαῖον, and at least on this point both seem to read a text which agrees literally with the Florentine recension.

[24] *Analytica Priora*, trans. Boethius, recensio Florentina [22, p. 26]. Both commentators read *deficit necessarium* with the Florentine rescension, whereas the Chartres rescension has *diminutum a necessario* [21, p. 160].

[25] Smith [28, p. 18] also translates διαλείπειν τὸ ἀναγκαῖον as "falls short of necessity"

on the reception of this passage among those who read it in Boethius's translation. Rather than suggesting that "men go grey" is a queer kind of necessity (one with "gaps", as Striker proposes [29, p. 131])[26] Boethius's text says rather unambiguously that it is *not* necessary. Readers who receive *Prior Analytics* I.13 in this translation therefore have an important interpretive parameter set for them: They must make sense of Aristotle's remarks, including Aristotle's invocation of necessity in the following lines, under the assumption that natural contingencies are *not* necessary.

It is therefore not surprising that the author of the 'Anonymus Aurelianensis III', the earliest known Latin commentary on the *Prior Analytics*,[27] takes for granted that the type of contingency exemplified here by "men go grey," "men grow" and "men shrink" are not necessities. He adds, however, that they are in fact also not really contingencies.[28] Instead, he claims, a so-called "natural" contingency is in fact "neither truly necessary nor truly contingent, but in a certain way intermediate".[29] That is, "natural contingency" is not a true type of contingency, but rather an intermediate modal status that Aristotle wishes to distinguish from *both* necessity and from contingency proper.

In the twentieth century, Albrecht Becker also defended the view that this passage is intended to distinguish "natural" contingencies from contingencies proper. This reading is however difficult to reconcile with

in his English translation, but it is not clear that διαλείπειν can have this meaning in Greek: See the entry in Liddell, Scott and Jones [17] and the extended case for a different translation in Striker [30].

[26] On Striker's interpretation, Aristotle's point is that, although it is not true that all men turn grey of necessity, "one might still say that they do age or turn grey of necessity if they live long enough" [29, p. 131]. Hence there is a necessity involved in the greying of men, albeit one with "gaps". We will see that Kilwardby's interpretation also makes use of the idea of an interruption to process that otherwise necessarily produces a certain outcome, but without assimilating natural contingency to necessity.

[27] Thörnqvist [36, p. 2] estimates a date of composition between 1160 and 1180. The commentary is probably based on a lost Greek model [8].

[28] *Sed nota contingens naturale non re uera esse contingens, sed sic dici, eo quod, cum neque sit necessarium neque contingens neque impossibile, aliquo tamen nomine oportuit ipsum appellari et propter affinitatem quidem, quam habet cum contingenti, appellatur contingens* [36, p. 101.7–10].

[29] *nec uere est contingens nec uere est necessarium, sed quodam modo medium* [36, p. 100.3–4].

the text.[30] These comments are, after all, explicitly framed as a discussion of the ways that "being possible" is said [29, I.13, 32b4–5],[31] where the context demands that we understand "possible" in the narrow sense of contingency. Aristotle does not deny, as we would expect him to on this reading, that natural contingencies are *contingent*; he rather says that they "have no continuous *necessity*" [29, I.13, 32b8].[32]

In any case, the 'Anonymus Aurelianensis III' does not tell us much about what this supposedly intermediate modal status consists in. The author does make the intriguing remark that indeterminate contingencies are said to be "indeterminate" because "they do not have a natural determinate end to which they are more related".[33] This suggests a teleological understanding of natural contingencies as descriptions of goal-directed occurrences. It also suggests that this is the sense in which they are related more "towards being." The anonymous commentator does not however develop these ideas further.

Robert Kilwardby also takes for granted that natural contingencies are not necessities. Unlike the author of the 'Anonymus Aurelianensis III', however, he emphasizes that natural contingencies are genuine contingencies, no less so than indeterminate contingencies.[34] Natural contingencies are, however, a special type of contingency for Kilwardby because the predication they express has a "natural cause".[35] What Kilwardby seems to mean is that, in a natural contingency, there is a causal link between the nature of the subject and predicate (for instance, between the nature of men and developing grey hair). This causal link is what privileges the occurrence of this contingent event over its non-

[30] Becker admits that on his reading the discussion is out of place and conjectures that 32b18–22 may be a later insertion [3, p. 77].

[31] τὸ ἐνδέχεσθαι κατὰ δύο λέγεται τρόπους. Boethius: *'contingere' duobus modis dicitur* [22, p. 27]. Cf. Striker [30, pp. 151–152n1].

[32] οὐ συνεχὲς μὲν ἔχει τὸ ἀναγκαῖον. Boethius: *non continuum quidem habet necessarium* [22, p. 27].

[33] *non habet secundum naturam certum finem, ad quem magis se habeat* [36, pp. 100.31–101.1].

[34] *tam contingens natum quam infinitum sit non necessarium et possit esse et non esse* [35, pt. 1, p. 396.545–546].

[35] *causam naturalem* [35, pt. 1, p. 370.129].

occurrence.[36] It is in other words natural for humans to develop grey hair because being human is the ultimate cause of grey hair.[37]

For Kilwardby, this raises the question why a natural contingency is not simply a necessity, given that the nature or essence of the subject causes it to have the property described in the predicate term. The question is especially pressing for Kilwardby, since he elsewhere defends an essentialist account of syllogistic necessity as reducing to one of Aristotle's first two modes of essential or "*per se*" predication.[38]

His way of dealing with this problem is to distinguish two readings of the predicate in a statement like "men go grey" and the modal statuses of the disambiguated predications. On the one hand, the predicate may be interpreted to denote the *process* of going grey.[39] On this reading, Kilwardby holds, the predication is not a contingency of any sort but rather a necessity since it is necessarily true that men are always in the *process* of going grey.[40] Since this is a necessity in a perfectly strict sense, it may even feature as a theorem of a demonstrative science. Kilwardby offers a chain of causes which suggest he would take the relevant demonstration to be as follows:[41]

1. Greying holds of the incorporation of phlegm into the upper part of the head,

[36] Kilwardby says that natural contingencies require a "cause dedicated more to one side [*causam ordinatam magis ad unam partem*, i.e., to the predication holding rather than not holding]" [35, pt. 1, p. 396.565–566].

[37] *Dicit igitur quod contingens prediffinitum quoddam est natum, scilicet quod habet causam naturalem ordinatam ad ipsum* [35, pt. 1, p. 370.128–131].

[38] "For necessary propositions reduce to some mode of *per se* inherence, following Aristotle's statement in *Posterior Analytics* I that 'Only *per se* inherences are necessary'." *Propositiones enim necessarie reducuntur ad aliquem modum inherendi per se, secundum quod dicit Aristoteles in primo Posteriorum, 'Sola per se inherentia sunt necessaria'.* [35, pt. 1, p. 160.458–461]. See Mendelsohn [20], Thom [33, pp. 108–125] and Thom [31, pp. 19–25].

[39] *motum in canitem* [35, pt. 1, p. 394.535].

[40] *Si dicat motum in canitiem sic semper ex necessitate canescit homo cum est* [35, pt. 1, p. 394.535–536].

[41] *Prouenit enim canities ex incorporatione fleumatis in superiori parte capitis, cuius incorporationis causa est diminutio caloris naturalis, et ista incorporatio et caloris diminutio semper fit et continue.* [35, pt. 1, p. 394.537–540].

2. the incorporation of phlegm into the upper part of the head holds of the loss of natural heat,
3. the loss of natural heat holds of all men.
4. Therefore, greying holds of all men.

This demonstration explains why men are necessarily in the process of going grey by recourse to their necessary loss of heat. This loss of natural heat takes the form of an incorporation of phlegm into the upper part of the head, which as such causes hair to become grey. The premises of the demonstration are meant to be *per se* predications, that is, necessary truths that hold on account of the essences of men and the various processes described (greying, losing natural heat, etc.). While it is strictly necessary for each of these *processes* to occur in men, it is not necessary for them to come to completion in men. Their completion is merely a frequent occurrence,[42] since the process of greying in men can be "obstructed."[43] The nature of men therefore does not *invariably* bring about the presence of grey hair [35, pt. 1, p. 370.131–135]; this only takes place given sufficient time for a man's hair colour to naturally fade, and a man might die before this occurs.[44]

For this reason, in order for "men go grey" to state a necessity, "grey" must be understood to denote the process of going grey, not the completion of this process.[45] If "grey" is understood to stand for the completion of the process of getting grey hair, then the statement "men go grey" is *not* necessary but only contingent [35, pt. 1, p. 394.540–541]. It is, however, a special type of contingency owing to its connection with this necessary process. Like a necessity, the contingent fact that men go grey derives ultimately from the natures or essences of men and going grey.

[42] *Si autem dicat terminum motus completum sic ut frequenter canescit homo.* [35, pt. 1, p. 394.540–541].

[43] *impediri* [35, pt. 1, p. 370.130].

[44] *causam ordinatam naturalem sed potest impediri quia non semper est homo* [35, pt. 1, p. 380.132–133]. The context indicates that *non semper est homo* should be taken here to mean that "a man does not exist [eternally]" rather than "there are not always any men", as Thom and Scott [35, pt. 1, p. 385] translate it. The relevant *dubium* is about whether the statement that a man goes grey requires a man to exist when he becomes grey. The doubt does not require entertaining the more remote possibility that all men should at some point cease to exist.

[45] *terminum motus* [35, pt. 1, p. 394.535].

It however "falls short of a necessity",[46] as Aristotle says on Boethius's translation, because the predicate is not guaranteed to hold of the subject by the causal link that exists between predicate and subject.[47]

Natural contingencies are thus, on Kilwardby's interpretation, predications where the predicate describes the completion state of a process that necessarily occurs in its subject but does not necessarily come to completion in its subject because the subject may not exist for long enough for this process to reach its natural culmination.[48] While in no way necessary, it follows on this analysis that every natural contingency is closely related to a necessity that may often even be expressed using the same terms. That is, while *actually becoming grey* is predicated only contingently of men, this contingency is natural because *undergoing the process which terminates in grey hair* holds necessarily of men.[49] It is Kilwardby's view that either the process or the completion may be understood by the predicate "goes grey"; consequently, either a necessity or a contingency may be understood by "men go grey", depending on

[46] *deficit a necessario* [35, pt. 1, p. 370.133–134].

[47] See further Thom [31, pp. 32–34] and Thom [34, pp. 150–152], which my reading so far stands in agreement with. Thom does not emphasize the distinction between process and completion readings of the predicate term as much as I think this should be emphasized, but my principal complaints about Thom pertain to his *formalization* of natural contingencies (discussed below), not his interpretation of what Kilwardby means by *contingens natum*.

[48] It is not clear that Aristotle takes the perishing of the subject to be the *only* way that a natural process can be obstructed. Striker [30, p. 158] proposes going bald as another way the process could be obstructed. It is hard to reconcile this with Kilwardby's view that the process of greying takes place of necessity as long as the subject is alive. Kilwardby in any case assumes that death is the central type of obstruction Aristotle has in mind [35, pt. 1, p. 380.132–133], and I will work with the assumption that all obstructions of natural processes take the form of their subject perishing in what follows.

[49] It is useful to compare Kilwardby's notion of natural contingency with what Freddoso [9, p. 225] calls a "deterministic natural tendency", at least as long as we bracket Freddoso's requirement that "only *free* causes can prevent deterministic natural tendencies from blossoming into full-blown natural necessities" (emphasis in original). Kilwardby does not seem to recognize any requirement that the impediment to a natural contingency always be a result of free agency, but he otherwise seems to conceive of the relationship between this type of modality and necessity in a way that is very close to Freddoso's analysis. I am grateful to Elena Baltuta for drawing my attention to Freddoso's work.

the reading given to the predicate term.

This account allows Kilwardby to avoid collapsing natural contingency into occurrence for the most part.[50] He may be contrasted in this respect with his contemporary Roger Bacon, who introduces "contingent in most cases"[51] and "naturally contingent"[52] as synonyms. Bacon draws a threefold distinction among contingencies between those which are "equally" related to being and non-being (like you sitting), those which occur in few cases (like discovering treasure while digging) and those which occur in most cases or "naturally" (like men going grey).[53] Bacon's scheme suggests a statistical, or proto-probabilistic understanding of natural contingency as a contingency which occurs frequently or with high probability.[54]

Kilwardby, by contrast, claims that there are occurrences which are natural in most cases (like men going grey in old age) as well as those which are natural in few cases (his example is men going grey in young age).[55] On Kilwardby's view, it is natural for men to go grey in old age because there is a necessary process in men that culminates in greyness. The fact that becoming grey in *old age* is natural in most cases merely marks the fact that this process typically takes a long time (in comparison with a man's lifetime) to complete. It is possible, if uncommon, for the process to come to completion early (and in that case greyness takes place in a young man) [35, pt. 1, p. 398.586–7] as well as for it not to come to completion within a man's lifetime at all (in which case it fails to take place in that man at all) [35, pt. 1, p. 370.132–4]. The relative rarity

[50] Cf. Thom [31, p. 32] contra Lagerlund [16, p. 45].

[51] *contingens ut in pluribus* [6, 2.1, §392].

[52] *contingens natum* [6, 2.1, §392].

[53] *Et illud contingens potest esse tripliciter: aut enim aequaliter se habet ad esse et ad non-esse, et tunc vocatur 'contingens ad utrumlibet' sive 'infinitum' sive 'aequale', ut 'Te sedere'; aut accidit in maiori parte, et tunc vocatur 'contingens ut in pluribus' sive 'contingens natum', ut 'Hominem canescere in senectute'; aut accidit in minori parte, et tunc dicitur contingens ut in paucioribus, ut 'Fodientes invenire thesaurum'* [6, 2.1, §392]. Note that Roger Bacon does not give this division in the course of commenting on *Prior Analytics* I.13. According to McCall [19, p. 68], a similar threefold distinction is found in Averroes.

[54] See Knuuttila [15, pp. 99–137] and Jacobi [13, pp. 18–20] for the statistical and proto-probabilistic interpretations respectively.

[55] Cf. Thom [34, p. 150].

of these outcomes does not however render them "unnatural": Kilwardby allows that in those cases where a man does go grey in youth, this is likewise a natural occurrence.[56] He thus preserves a distinction between the merely statistical notion of occurrence in most cases or "for the most part", and a separate, orthogonal notion of a natural occurrence.[57]

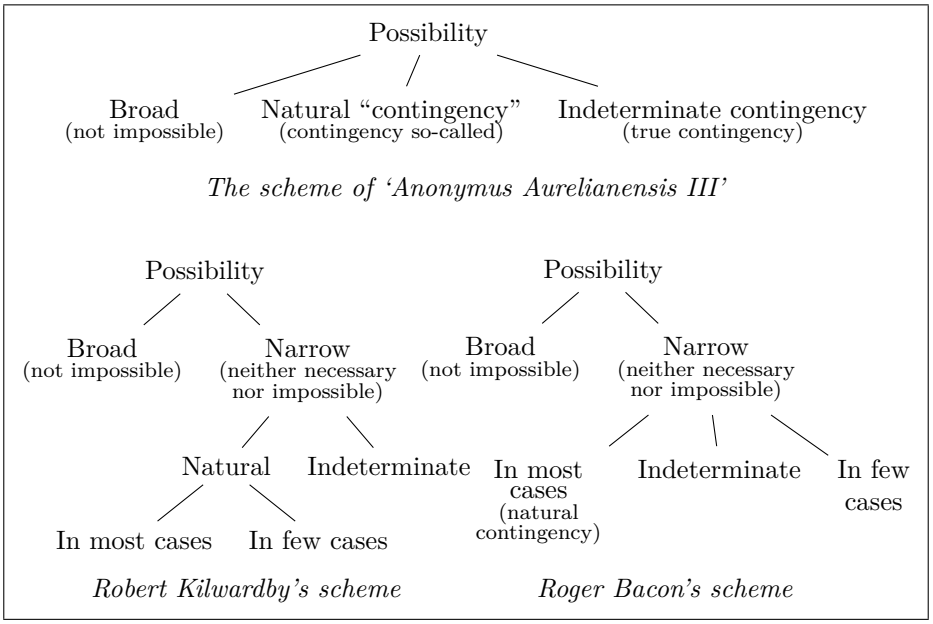

Figure 1: Divisions of possibility

These interpretive manoeuvres allow Kilwardby to make sense of Aristotle's remarks concerning natural contingency as he receives them. Kilwardby's denial that any contingencies, natural or not, serve as scientific premises, agrees with Aristotle's claim in the *Posterior Analytics* that the premises of demonstrations are necessities.[58] At the same time, he is able to make sense of Aristotle's statement in *Prior Analytics* I.13

[56] *canescere in iuuentute quod est natum in paucioribus* [35, pt. 1, p. 398.586–7].

[57] For this reason the statistical interpretation of natural contingency presented in Knuuttila [15, pp. 99–137] and Jacobi [13, pp. 18–20] does not apply to Kilwardby. Thom [31, p. 32] rightly objects to the attribution of this view to Kilwardby in Lagerlund [16, p. 45].

[58] *Posterior Analytics* I.4, 73a21–24. For Kilwardby's commentary on this chapter, see Cannone [5].

that "by and large arguments and investigations are concerned with what is possible in this way" (i.e., naturally contingent) [29, I.13, 32b20–21].[59] These words come to Kilwardby in Boethius's translation as *et paene orationes et considerationes fiunt de sic contingentibus* [35, pt. 1, p. 372.178–9]. Kilwardby understands *paene* here in the sense of "roughly speaking".[60] According to Kilwardby, Aristotle employs the qualification "roughly speaking" because "there are demonstrations about such things, not insofar as it is possible for them not to be so, but insofar as they are necessary" [35, pt. 1, p. 372.165–167]. What Kilwardby seems to mean is that, although Aristotle does not in fact hold that there are demonstrations of natural contingencies, he does hold that there are demonstrations of closely related, necessary propositions, namely of the predications describing the processes underlying natural contingencies. Such necessities may, as in the case of "men go grey", even be expressed using the same terms as a natural contingency. We can thus understand why Aristotle might have said that there is "roughly speaking" a demonstration containing the natural contingency: There is, at any rate, a demonstration of a closely related statement (in this case, that men undergo the process of greying), and the syllogistic argument for a natural contingency will employ the same terms as this demonstration, albeit with their completion readings intended.

Kilwardby's distinction between process and completion readings of predicates also allows him to make sense of Aristotle's remark that a natural contingency "has no continuous necessity because a man does not exist forever, but while a man exists, it happens either of necessity or for the most part" [29, I.13, 32b8–10].[61] On Kilwardby's reading, the reason that it is merely contingent for men to go grey, rather than necessary, is because the process of greying is not "continuous": It may be "interrupted", in particular when a man dies.[62] Thus, on Kilwardby's

[59] σχεδὸν οἱ λόγοι καὶ αἱ σκέψεις γίνονται περὶ τῶν οὕτως ἐνδεχομένων

[60] This is how Thom and Scott [35, pt. 1, p. 373], correctly in my view, render the term in Kilwardby's commentary.

[61] τοῦτο γὰρ οὐ συνεχὲς μὲν ἔχει τὸ ἀναγκαῖον διὰ τὸ μὴ ἀεὶ εἶναι ἄνθρωπον, ὄντος μέντοι ἀνθρώπου ἢ ἐξ ἀνάγκης ἢ ὡς ἐπὶ τὸ πολύ ἐστιν. Boethius: *hoc enim non continuum quidem habet necessarium eo quod non semper est homo, cum autem homo est aut ex necessitate aut ut in pluribus est* [22, p. 27].

[62] *hominem canescere non habere continuam necessitatem eo quod non semper est*

reading, Aristotle is saying that, while it is indeed necessary for a man to be *going grey* so long as that man exists, the necessity is not "continuous" for the reason that each man only has a finite lifetime and thus the greying process may not finish within the man's lifetime. As for Aristotle's next remark, that greying happens "either of necessity or for the most part" during a man's life, Kilwardby takes this to be an allusion to the distinction between process and completion readings of natural predicates [35, pt. 1, p. 394.530–542]. If "go grey" is understood to mean "is in the process of going grey", then it is necessary for men to be going grey (notwithstanding the fact that this necessity is "non-continuous" owing to the finite lifespan of men). On the other hand if "go grey" is understood to mean "reaches the stage of actually having grey hair", then this is true merely "for the most part" [35, pt. 1, p. 394.540–541].

Kilwardby is thus able to give answers to the most significant interpretive difficulties this passage presents. It does not follow that Kilwardby's interpretation is superior to modern interpretations of Aristotle that find no answers to some of these questions in Aristotle's text, since it of course remains a possibility that Aristotle's own theory of natural contingency was not fully developed. The interpretive advantage of coherence must be weighed against the interpretive cost of attributing to Aristotle distinctions which are not to be found in the text. In particular, as we have seen, Kilwardby's reading turns on a distinction between process and completion readings of terms, and for this distinction Kilwardby does not refer us, as he often does, to any other text in the Aristotelian corpus. Further, as we have seen, Kilwardby reads Aristotle in Boethius's translation, and it is not clear that Boethius's translation is a faithful rendering of Aristotle's Greek here. For these reasons, the notion of natural contingency Kilwardby develops in his commentary on *Prior Analytics* I.13, 32b4–22 is still best viewed as a development of Aristotle's ideas rather than simply an exposition of them; nevertheless, it is a testament to Kilwardby's prowess as an interpreter that he develops a view of natural contingency that is consistent with Aristotle's exact words as he receives them.

homo [35, pt. 1, p. 384.381–2].

2 Kilwardby on the logic of natural contingency

Kilwardby employs the interpretation he develops of the meaning of natural contingency and the conditions under which natural contingencies hold as the basis for his remarks about their logic. His remarks can be divided into two classes: Those about the conversion rules for natural contingencies and those which concern the syllogistic results involving natural contingency as a special modality.

2.1 Conversion

Aristotle does not say much about the logic of natural contingency. The only claim he makes about the logic of natural contingency comes in the course of introducing the distinction between natural and indeterminate contingency, where Aristotle claims that both natural and indeterminate contingencies "convert with respect to opposite premisses" [29, I.13, 32b13–14].[63] This is the language Aristotle uses to describe the rule of "complementary conversion"[64] above: That is, the rule that if it is contingent that all As are B, then it is contingent that no As are B, and if it is contingent that some As are B, then it is contingent that not all As are B [29, I.13, 32a31–32].

These rules are reasonable if contingency is understood in the broad sense of neither necessary nor impossible. Yet, as contemporary scholars have pointed out, this rule is invalid if "contingent" means "for the most part" or means "occurs by nature": From the fact that men for the most part go grey it does not follow that, for the most part, men do not go grey; and from the fact that it is natural for men to go grey, it does not follow that it is natural for them not to do so [see 30, p. 149, 2, p. 351].

Kilwardby, like other commentators around his time,[65] recognizes the problem with taking natural contingency to obey this rule. As he says, natural contingencies "have a cause that is dedicated to one side rather

[63] ἀντιστρέφει μὲν οὖν καὶ κατὰ τὰς ἀντικειμένας προτάσεις.

[64] For this terminology see Ross [26, p. 45].

[65] See Albert the Great [4, tract. I, c. 12–14] and, especially, the extended treatment of conversion in Lambert of Auxerre [1, pp. 41–49]. Conversion is also a focus in the 'Anonymus Aurelianensis III' [36, pp. 97–101].

than the other — on account of which, the negation does not have such a cause."[66] If the essence of man entails that men are necessarily in the process of greying, then, assuming the essence of man is consistent, it cannot also entail that men are not in the process of greying [35, pt. 1, p. 396.565–7]. Hence, the fact that it is naturally contingent for all men to go grey cannot imply that it is naturally contingent for no men to do so. Consequently the law of complementary conversion is invalid for natural contingency: $AQ_{nat}aB$ does not imply $AQ_{nat}eB$, and $AQ_{nat}iB$ does not imply $AQ_{nat}oB$.[67]

Kilwardby reconciles this observation with the text by denying that Aristotle meant to state that natural contingencies convert with natural contingencies. Instead, he understands Aristotle's remark that natural contingencies "convert with respect to opposite premisses, but not in the same way" to mean that a natural contingency converts to a possibility in the broad sense [35, pt. 1, pp. 370.149–157, 396.562–3]: If it is naturally contingent for all men to go grey, then it is possible (in the broad sense of not impossible) for none to go grey. In other words, if we write 'M' for possibility in the broad sense, then Kilwardby accepts the rule that $\mathrm{Q}_{\mathrm{nat}}$a converts to Me (and $\mathrm{Q}_{\mathrm{nat}}$i with Mo).[68] Kilwardby notes that this fits with his view of natural contingency as having a cause that may be *impeded*: If it is possible, say, for all men to become musical, and this expresses a natural contingency, then the converted proposition that it is

[66] *habet causam ordinatam magis ad unam partem, propter quod eius negatio non habet causam talem* [35, pt. 1, p. 396.565–7].

[67] Here and throughout I adopt the notation for syllogistic premises and conclusions used by Thom [34], adapted from the notation that has become standard in studies of ancient and medieval syllogistics since McCall [19]. Modalities are signified by L (necessity), M (possibility), Q (contingency) and X (assertoric); I introduce $\mathrm{Q}_{\mathrm{nat}}$ (natural contingency) and $\mathrm{X}_{\mathrm{simp}}$ (temporally unrestricted assertoric). Quantities and qualities of propositions are signified by 'a' (universal affirmative), 'e' (universal negative), 'i' (particular affirmative) and 'o' (particular negative). In writing syllogistic premises and conclusions using this notation, I place the predicate first. Thus "$ALeB$" is to be read as "A holds necessarily of no B", etc. I also use the standard medieval mnemonic names for syllogisms [on which see 24] followed by a string of modalities in order to denote the modality of their premises and conclusion in a modal syllogism. Thus, "Barbara LXL" refers to a Barbara syllogism with a necessity major premise and an assertoric minor, i.e. the argument $ALaB$, $BaC \vDash ALaC$.

[68] See note 67 for a fuller description of the notation used here and throughout.

possible for no men to become musicians is true only because the natural process of musical education can be impeded, not because it is natural for them to fail to do so [35, pt. 1, pp. 154.379–156.392].[69]

2.2 Syllogisms

Unlike Aristotle, who leaves aside the notion of natural contingency after introducing it in the passage discussed above,[70] Kilwardby makes judgements about the role that natural contingencies play in syllogistic modal logic. He does not however systematically present an account of their logic. Following the order of Aristotle's exposition, he makes scattered judgements about the cases in which the contingency premises of a syllogism need to be interpreted as natural rather than indeterminate or generic contingencies. Despite lacking any formal exposition of the logic of natural contingency, however, these judgements can be seen to cohere with his theory of the meaning of "contingent" in this sense.

[69] Paul Thom has pointed out to me that assimilating the case of learning music to the same class of contingencies as men going grey risks trivializing the notion of a natural contingency, since this example then requires us to understand the class of things which may "impede" becoming a musician very broadly, as including anything except the class of events where a man seeks to become a musician and undergoes the necessary training. The worry is then that we might as well say that any contingent predication is natural (it's naturally contingent, e.g., for men to become astronauts since there are specific circumstances under which they do so). One possible solution is to take Kilwardby to recognize two classes of natural contingencies, one exemplified by "men go grey in old age" and another by "men become musicians". In that case, my analysis in this paper should be taken to cover only natural contingencies of the first kind. As against this reading, however, Kilwardby gives no indication that he intends only a partial analysis of natural contingencies when he focuses on the example of "men go grey in old age" in his commentary on *Pr. An.* I.13. Further, Kilwardby does say that humans have a "natural power" to become musical [35, pt. 1 p. 154.385], and that there is a "dedicated cause" for the predicate musical to hold of human [35, pt. 1 p. 156.388]. This suggests that Kilwardby might take musical skill to be the normal actualization of a human ability, a skill which humans will inevitably develop given the desire and opportunity to do so. The same could not be said of, e.g., discovering treasure while digging or becoming pale upon illness. In that case, the class of natural contingencies will turn out to be broader than we might first have expected, but not trivially broad. Whether becoming an astronaut will count as a natural contingency will depend on whether we take humans to have natural abilities that normally result in becoming astronauts under the appropriate circumstances.

[70] See Becker [3, p. 77]; Striker [30, p. 150].

The first set of judgements Kilwardby makes about the syllogistic logic of natural contingencies concerns syllogisms in the first figure with premises of mixed modality. Kilwardby denies that the contingency needs to be understood as a natural contingency in first figure syllogisms with an assertoric major and a contingency minor [35, pt. 1, pp. 458.459–464.542 (*dub.* 6)]. Instead, the contingency premise is to be understood here in the broad sense that includes both indeterminate and natural two-sided contingencies. In other words, Kilwardby claims that Barbara XQM and Celarent XQM are both valid without any special restriction to natural contingencies [35, pt. 1, p. 462.512–514].[71] On the other hand, he maintains that in Celarent and Ferio with a necessity major and a contingency minor, the contingency must be understood as a natural contingency [35, pt. 1, pp. 500.133–137, 516.419–422, 526.561–528.593 (*dub.* 10)].

He takes the contingency in these syllogisms to be a natural contingency in order to block specious counter-examples to these moods. In first-figure syllogisms with a necessity major and an assertoric minor, Kilwardby had argued that the minor premise must be true without temporal restriction (*simpliciter*), that is, it must hold of all times and not merely "as of now" (*ut nunc*) [35, pt. 1, pp. 316.149–318.163]. This distinction recalls to Aristotle's remark in *Prior Analytics* I.15 that the universal assertoric premise of Barbara XQM must be taken "not with a limitation of time such as 'now' or 'at such-and-such a time', but without qualification" [29, I.15, 34b7–8].[72] While the role this clarification is meant to play in Aristotle's own treatment of syllogisms is not entirely clear,[73] Kilwardby employs this distinction primarily in order to dismiss apparent counterexamples to syllogisms that contain one assertoric and one modal premise. For example Kilwardby observes that Barbara LXL, a mood Aristotle endorses, suffers from this seeming counterexample:

> Of necessity every man is an animal;
> everything white is a man (let it be so);
> but not of necessity everything white is an animal [35, pt. 1,

[71] See further Thom [33, pp. 85–88]. On the notation used here, see note 67.

[72] μὴ κατὰ χρόνον ὁρίσαντας, οἷον νῦν ἢ ἐν τῷδε τῷ χρόνῳ, ἀλλ' ἁπλῶς

[73] For a different view, which does not take *simpliciter* assertorics to hold at all times, see Malink [18, pp. 234–237].

> p. 322.235–237 (*dub.* 7)]

His response is to deny that the premise "everything white is a man" is an assertoric statement of the right kind: This is merely true "as of now", whereas premises in the modal syllogistic ought to be true "*simpliciter*".

Kilwardby describes this restriction in terms of the "power" of the major premise to "appropriate" the minor so as to be a *simpliciter* assertoric [35, pt. 1, p. 322.232–274].[74] He argues that in Celarent with a necessity major and a contingency minor, the necessity major similarly "appropriates the minor to itself so that it is a natural contingency".[75] This restriction is necessary, since Celarent LQX suffers from the following counter-example:

> of necessity no stone is a man;
> it's contingent for everything moving to be a stone,
> but it's not the case that nothing moving is a man [35, pt. 1, p. 512.331–333 (*dub.* 5)]

This is an instance of Celarent LQX, a mood Aristotle apparently endorses at *Prior Analytics* I.16, 36a7. Yet while the premises of this argument are true, the conclusion need not be. Kilwardby's response is to add the stipulation that the minor premise be a natural contingency [35, pt. 1, p.516.419–422 (*dub.* 5)]. An unrestricted assertoric conclusion *does* follow from a universal negative necessity and a universal affirmative natural contingency, as Kilwardby illustrates using the following example:

> Of necessity no musician is a log;
> it's contingent for every man to be a musician;
> so no man is a log [35, pt. 1, p. 528.589–591 (*dub.* 10)]

Here "every man is a musician" is taken to be a natural contingency, presumably with the idea that humans naturally develop a propensity for music so long as nothing impedes this development [35, pt. 1, pp. 154.385–156.386].[76] He also gives the following argument for the validity of this mood; however, the argument does little more than to re-state his

[74] For a discussion of Kilwardby's doctrine of appropriation as it relates to mixed necessity syllogisms, see Mendelsohn [20].

[75] *appropriat sibi minorem ut sit de contingenti nato* [35, pt. 1, p.526.582].

[76] On becoming a musician as a natural contingency see note 69.

definition of a natural contingency as a contingency with a dedicated cause:

> The reason is also clear as follows, that if it's impossible that some B is A when B contingently inheres in C according to a cause innate to it so that the minor is a natural contingency, it's clear that C will not be able to be A. And so there will be an unqualified assertoric conclusion of necessity, as is clear from the terms mentioned ('log, musician, man') [35, pt. 1, p. 530.620–625 (*dub.* 12)].

Kilwardby thus concludes that the syllogism Aristotle perfects, and which should be understood to exist on the basis of *Prior Analytics* 36a7–10, is Celarent $\mathrm{LQ_{nat}X_{simp}}$. He alludes to, but does not spell out, an analogous argument intended to show, *mutatis mutandis*, that in Ferio with a necessity major and contingency minor, the minor premise must be taken to be a natural contingency (since this syllogism is perfected by Aristotle, on his interpretation, from Celarent $\mathrm{LX_{simp}L}$).[77] And he gives an argument along similar lines for the conclusion that the Cesare syllogism with a necessity major and a contingency minor that Aristotle intends in *Prior Analytics* I.19 (38a16–21) is Cesare $\mathrm{LQ_{nat}X_{simp}}$ (rather than Cesare LQX, $\mathrm{LQ_{ind}X}$, etc.) [35, pt. 1, pp. 586.221–588.236]. It will be my object below, in formalizing Kilwardby's logic of natural contingencies, to show why these results do indeed follow given the way Kilwardby understands the meaning of natural contingency.

[77] See [35, pt. 1, p. 500.135–142]; and cf. [35, pt. 1, pp. 516.423–426, 528.592–3] (note the plural *modis*). Kilwardby also claims in passing that the premises *ALeB*, *BQoC* will yield the same conclusion as Ferio with a necessity major and a contingency minor since the minor premise can be converted "by opposite qualities" (i.e., complementary conversion [35, pt. 1, p. 500.142–146]). Here he seems either to forget his restriction of the Ferio syllogism to a natural contingency, or else to forget that he denies complementary conversion holds of natural contingencies. I will disregard this validity claim in what follows.

3 Formalizing Kilwardby's logic of natural contingency

The task of formalizing Kilwardby's logic in its entirety has been undertaken by Paul Thom [34]. This study goes beyond his previous work on Kilwardby's logic[78] in providing a model-theoretic semantics to capture the meaning Kilwardby takes syllogistic sentences to have, and meticulous accompanying proofs to show that this semantics captures Kilwardby's logical arguments and results. It is shown in detail how the background assumptions at work in Kilwardby's logic are stated and expounded in other works, such as Kilwardby's commentaries on Porphyry's *Eisagōgē*, *De Interpretatione* and the *Categories*. There are few parallels one can point to in contemporary scholarship on the history of logic that so adeptly combine first-rate textual scholarship with technical sophistication.

Following Kilwardby's claim in his commentary on the *De Interpretatione* and the *Prior Analytics* that terms correspond to concepts (*intellectus* or *rationes*), Thom interprets the sentences of Kilwardby's syllogistic logic in models that comprise a field of concepts ordered by various primitive intensional relations. In addition to a domain of individuals $\mathbf{D}$ and an associated interpretation function $\mathbf{I_D}$, Thom's models thus also include a function $\mathbf{I_F}$ which assigns terms of the language to elements of its conceptual field $\mathbf{F}$. Some of these concepts are distinguished as *essences* (genera and species belonging to the category of substance), denoted by a set $\mathbf{E}$. The field of concepts is structured by primitive relations of inseparability ($\Leftarrow$), essential inseparability ($\Leftarrow_{\mathrm{E}}$, which requires the subject to be an essence term), repugnance ($\Downarrow$) and denomination (AB). Collecting these elements into a tuple, we may identify a model with a structure

$$\mathcal{M} = \langle \mathbf{D}, \mathbf{I_D}, \mathbf{F}, \mathbf{E}, \mathbf{I_F}, \mathbf{R} \rangle$$

where $\mathbf{R}$ is a structure interpreting the primitive conceptual relations ($\Leftarrow$, $\Leftarrow_{\mathrm{E}}$, $\Downarrow$, AB).

Necessities and contingencies are interpreted in terms of these primitive

[78] See especially Thom [32] and Thom [31].

relations of repugnance and (essential) inseparability. It will not be necessary to go into the details of Thom's formalization of necessity and generic contingency and other modalities here.[79] The only important point, for our purposes, is that their truth conditions are given entirely in terms of $\mathbf{R}$ and $\mathbf{I_F}$. The domain of individuals and its associated interpretation function plays no direct role in their interpretation on Thom's reconstruction, making this an "intensional" interpretation of Kilwardby's modal logic.

Thom convincingly shows that this aligns with the way Kilwardby understands necessities and generic contingencies in syllogisms. However, this intensional framework is less well suited to capture Kilwardby's notion of a *simpliciter* assertoric. As we have seen, Kilwardby contrasts "as of now (*ut nunc*)" assertorics with *simpliciter* assertorics which are taken to hold at all times. As Thom interprets the distinction, the truth conditions of *ut nunc* assertorics are extensional (given in terms of $\mathbf{D}$ and $\mathbf{I_D}$), whereas the truth conditions of *simpliciter* assertorics are intensional (given in terms of $\mathbf{F}$ and $\mathbf{I_F}$). In particular, a universal affirmative *ut nunc* assertoric is true just if the subject term's extension is non-empty and included in the extension of the predicate term:

AaB is true in $\mathcal{M}$ iff $\mathbf{I_D}(B) \neq \varnothing \wedge \mathbf{I_D}(B) \subseteq \mathbf{I_D}A$

The corresponding *simpliciter* proposition, on the other hand, says on Thom's semantics that the concept associated with the subject term bears the intensional relation of inseparability to the predicate:

$AXaB$ is true in $\mathcal{M}$ iff $\mathbf{I_F}(A) \Leftarrow \mathbf{I_F}(B)$.[80]

Similarly, a universal negative assertoric says that the extensions of the two terms have an empty intersection, while the *simpliciter* assertoric says, in Thom's formalization, that the concept associated with the subject term bears the intensional incompatibility relation to the concept associated with the predicate term:

[79] For these, see Thom [33, chs. 5 and 6].

[80] Like Thom [33], I use X to indicate a modality of *simpliciter assertoric* in a proposition (like AXaB), but to indicate regular assertoric in the name of a syllogism (like Barbara LXL). I use $\mathrm{X}_{\mathrm{simp}}$ in the name of the syllogism to indicate that a premise is a simpliciter assertoric.

AeB is true in $\mathcal{M}$ iff $\mathbf{I_D}(B) \cap \mathbf{I_D}(A) = \varnothing$

$AXeB$ is true in $\mathcal{M}$ iff $\mathbf{I_F}(A) \Downarrow \mathbf{I_F}(B)$

The truth conditions of particular assertorics are the negations of the propositions of opposite quality, guaranteeing Aristotle's square of opposition for both *ut nunc* and *simpliciter* assertorics. In essence, the difference between the *ut nunc* and *simpliciter* truth conditions as formalized by Thom are thus (i) that the truth condition for an *ut nunc* assertoric is formulated in terms of extensional relations of set inclusion, exclusion and non-emptiness whereas the truth condition for a *simpliciter* assertoric is formulated in terms of Thom's special intensional relations, and (ii) the truth condition for the *ut nunc* assertoric is formulated in terms of the extensional interpretation function $\mathbf{I_D}$ (which, recall, maps to sets of individuals from the domain), whereas the *simpliciter* assertoric is formulated in terms of the intensional interpretation function $\mathbf{I_F}$ (whose values are concepts from the field of concepts $\mathbf{F}$).

What is noteworthy about this analysis for our purposes is that temporal notions play no role in it. This is perhaps surprising, since Kilwardby's distinction between *simpliciter* and *ut nunc* assertorics is quite explicitly a temporal one. In his commentary on *De Interpretatione*, Kilwardby glosses the distinction as follows:

> Something is taken to be *simpliciter* when something is predicated perpetually and incorruptibly, as in "man is an animal"; something is taken to be according to a time when something is predicated in time and variably, as "man is white".[81]

Kilwardby is contrasting predications which hold at some given time with those that hold at all times.[82] Now, it is plausibly a *sufficient*

[81] Robert Kilwardby, *Notulae super librum Perihermeneias*, pt. 1, L.2 (*ad* 16a18): *esse simpliciter opponitur quando praedicatur aliquid perpetuum et incorruptibile, ut 'Homo est animal'; secundum tempus quando predicatur aliquid temporale et variabile, ut 'Homo est albus'*, quoted in Thom [33, p. 82n17]. My translation.

[82] I grant that the term "incorruptibly" (*incorruptibile*) suggests an intensional dimension to his understanding of the distinction as well. This is not surprising, since Kilwardby takes *simpliciter* assertorics to imply certain intensional conditions (see below). Nevertheless, it seems clear that the basic distinction here is between predications true at a particular time and those true throughout all time (the latter

condition for an extensional predication to hold at any given time that a corresponding predication among concepts hold; at any rate, this is something Kilwardby seems to assume.[83] Further, Kilwardby also seems to hold that a *simpliciter* predication *entails* a relation of inseparability or repugnance between the concepts of the terms involved.[84] Hence, an intensional relation between terms does seem to be a necessary and sufficient condition for Kilwardby's notion of a *simpliciter* assertoric. However, that still does not mean that Kilwardby's distinction between *ut nunc* and *simpliciter* assertorics should be identified with the distinction between intensional and extensional predication, as it is in Thom's formalization. A more faithful representation of Kilwardby's position would introduce times into the models used to interpret syllogistic statements. We could then introduce bridging principles to guarantee that omnitemporal relations among extensions correspond with intensional relations among concepts.

I sketch such an alternative formalization below. This would be a pedantic complaint if *simpliciter* assertorics were only ever used in Kilwardby's logic in order to infer an intensional state of affairs from which further consequences may be derived. In that case, while the formalization proposed would still be closer to Kilwardby's own presentation, it would shed little light on the workings of Kilwardby's actual logic. The success of Thom's system in capturing Kilwardby's necessity and (generic) contingency syllogistic shows that this is indeed the *main* way that Kilwardby uses the notion of a *simpliciter* assertoric. However, this failure to take

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also being "incorruptible").

[83]For this assumption in Kilwardby, see Thom [33, pp. 88–90, 99] (especially postulates 4.1 and 4.2).

[84]See for instance [35, pt. 1, pp. 389–391], where Kilwardby claims that a *simpliciter* assertoric requires "that the concepts of the terms cohere or conflict (*quod rationes terminorum cohereant uel discohereant*)". Unlike Thom [33, pp. 88–89], I don't think this is meant by Kilwardby as a general definition of *simpliciter* predication. Kilwardby's point here is that *simpliciter* predication is not necessity in the *per se* sense required for necessary premises and conclusions in the modal syllogistic, *even though* they do entail a condition that might easily be confused with *per se* necessity. The assumption that a *simpliciter* predication implies an intensional relation among concepts is also evident in Kilwardby's association of *simpliciter* assertorics with habitudinal predication, which he does seem to take to be an intensional relation: See [35, pt. 1, p. 458.445–450] with Thom [33, p. 83].

into account temporal notions in Kilwardby's logic becomes important when we try to interpret what Kilwardby has to say regarding *natural* contingency, as we will see.

3.1 Thom's formalization of Kilwardby on natural contingency

Let us begin with Thom's analysis of *potentiality*, which plays a key role in his analysis of natural contingencies (but not contingencies of other types). Thom includes in his language an operator on terms that produces, for every term A, a term A_p to be read as "the potential for A". Potentialities are governed by the sole axiom

> If $\lambda \Leftarrow \mathbf{I_F}(A)$ and $\mathbf{I_F}(A_p) \Leftarrow \kappa$ and $\kappa \in \mathbf{E}$ then $\neg\lambda \Downarrow \kappa$ [34, p. 74]

That is, if something is inseparable from the actualization of a potentiality, and this potentiality is in turn inseparable from some essence, then that first thing must not be repugnant to that essence. This condition seems intended to capture Kilwardby's idea that potentialities, at least of the sort that syllogistic propositions concern, must be realizable: There can be no concepts which exist *merely potentially* but could never be realized.[85] Thom captures this idea in the above condition by stipulating that if a potentiality is inseparable from an essence concept (if, say, the capacity to become educated is inseparable from the concept human) then there cannot be a concept inseparable from the actualization of that potentiality (for instance, in this case, being literate) which is incompatible with the underlying essence kind. In other words, potentialities are never inseparable from concepts that would inherently preclude their actualization in an essence from which they are inseparable.

This notion of a potentiality forms the basis for Thom's interpretation of Kilwardby's natural contingencies. Thom only attempts to formalize universal affirmative natural contingencies, even though Kilwardby seems to allow particular affirmative necessities as well.[86] He proposes the

[85] Or if there are, they are not the concern of syllogistic logic: Cf. Thom [33, pp. 67–68].

[86] Kilwardby explicitly denies that there are negative natural contingencies [35, pt.

following truth condition:

> $AQ_{nat}aB$ is true iff $AQaB$ is true and being A_p is inseparable from B, where B is an essence and A is a denominative [33, p. 152]

A universal affirmative natural contingency statement thus expresses the same as a regular (unampliated) universal affirmative contingency with the additional requirements that (i) the subject is an essence term and the predicate is a (substantivized) accident term (or "denominative"), and (ii) the *potentiality* to have the property expressed by the predicate is inseparable from the subject. Putting aside these restrictions on term kinds, this is to say that A holds naturally contingently of all B ($AQ_{nat}aB$) if, and only if (i) the contingency $AQaB$ holds, and (ii) being potentially A is inseparable from B.

This no doubt captures some aspects of what Kilwardby takes to be entailed by a natural contingency. A natural contingency is non-accidental: This is captured by the second clause, which requires that the potentiality be inseparable from the subject term. Kilwardby also takes natural contingencies to imply contingencies in the generic sense, which is ensured by the first clause. And Kilwardby does seem to take all naturally contingent properties to be at least possibly realizable, which is captured by Thom's axiom regarding potentialities.

This is, then, plausibly a necessary condition for natural contingency as Kilwardby understands it. Yet it is clearly not a sufficient condition: Beyond saying that a regular contingency is true, it says only that the potentiality associated with the predicate term is inseparable from the subject and must be realizable within it, and restricts the types of the relevant terms. Yet, as we have seen, Kilwardby holds something much stronger than this concerning natural contingencies. He claims not just that a naturally contingent property *can* always be realized in its subject; he holds that a natural contingency *always will* be realized in its subject so long as nothing obstructs its realization. For Kilwardby,

1, p.398.578–595], but he presupposes that there are particular affirmative natural contingencies when he endorses the validity of Ferio $\mathrm{LQ_{nat}X_{simp}}$ [35, pt. 1, pp. 526.561–528.593 (*dub.* 10)].

the distinguishing feature of a natural contingency is that it describes a causal process which *occurs of necessity*[87] and as such *will necessarily terminate in actuality, given sufficient time to do so before the subject in which it occurs expires.*

This central element of Kilwardby's account is not captured by Thom's definition. Further, it is difficult to see how this could be captured in Thom's framework, since he takes the truth conditions of modal syllogistic propositions to be grounded in timeless relationships between concepts, whereas Kilwardby's analysis makes reference to individuals, time, and processes which occur in them over time.

In addition to these problems capturing Kilwardby's metaphysical analysis of natural contingency, it is also questionable whether Thom's interpretation adequately captures the syllogistic results involving natural contingency that Kilwardby sets out. As we have seen, Kilwardby holds that in a Celarent syllogism with a necessity major and a contingency minor, the minor must be a natural contingency in order to yield a *simpliciter* assertoric conclusion. As Thom interprets Kilwardby here, the conclusion of the Celarent syllogism must be interpreted not as an unrestricted assertoric with its usual semantics but rather as one whose meaning is the same as a (broad) possibility proposition, since the mood is reduced from Ferio LXL where the minor is a *simpliciter* assertoric. Thom introduces the notation $\mathrm{X_M}$ to mark this, and writes the relevant syllogism as Celarent $\mathrm{LQ_{nat}X_M}$. But Kilwardby does not, as far as I can see, make the claim that the conclusion here is to be read merely as a possibility. He claims only that the conclusion is to be read as an unrestricted assertoric [35, pt. 1, pp. 516.423–424, 527.578–528.588]. And, besides, the inference with the stronger conclusion is intuitively valid, as Kilwardby points out [35, pt. 1, p.528.589–591 (*dub.* 10), discussed above]. If it is of necessity the case that nothing which develops grey hair is a log, and if it is naturally contingent for all men to develop grey hair, then it is always actually the case that no men are logs (and not merely possibly).

[87] See [35, pt. 1, pp. 535–6], where "goes grey" is used as the example of the natural predicate: "If [the act of going grey] bespeaks the process of going grey then when there are men they are always and of necessity going grey [*si dicat motum in canitiem sic semper ex necessitate canescit homo cum est*]".

There are, then, three problems with Thom's formalization of natural contingency in Kilwardby. First, it fails to capture the metaphysics of natural contingency as Kilwardby understands it, in part because it fails to formalize Kilwardby's use of temporal and extensional notions in his account of what a natural contingency means. Thom claims that this metaphysical level of analysis is irrelevant to logic for Kilwardby,[88] but it is not clear why this should be so.[89] Second, Thom's formalization does not capture the central syllogistic result that Kilwardby endorses in connection with natural contingency (namely Celarent $\mathrm{LQ}_{\mathrm{nat}}\mathrm{X}_{\mathrm{simp}}$). Finally, there is the more minor problem that Thom's analysis fails to account for particular affirmative natural contingencies, which Kilwardby does seem to at least implicitly recognize. In the next section, I will consider in what way Thom's semantics must be enriched in order to more satisfactorily incorporate Kilwardby's analysis of natural contingency.

3.2 An alternative formalization of Kilwardby on natural contingency

How can we formalize Kilwardby's notion of natural contingency in a way that hews more closely to the meaning of natural contingency as Kilwardby understands it? As we have seen, Kilwardby's notion of natural contingency makes reference to individuals and to time. What makes the contingency that men go grey distinctively *natural*, for Kilwardby, is the fact that any particular man will eventually get grey hair if that man lives long enough.

It is not obvious how to formalize this condition in contemporary modal logic.[90] We will proceed by taking Kilwardby's statements about natural contingency as the basis for a model-theoretic semantics, using Thom's models as a basis. We expand these models to include a timeline $\mathbf{T}$. For simplicity, we assume that time is non-branching and thus that $\mathbf{T}$ is a

[88] "the science of logic does not need to refer to this metaphysical grounding" [33, p. 151].

[89] At any rate, Thom does not seem to have any qualms about referring to the metaphysical grounding of logic in order to develop the semantics for Kilwardby's logic of necessities, as we have seen.

[90] I am grateful to Stephen Read for drawing my attention to some of the difficulties involved. I provide a tentative suggestion in my concluding remarks below.

segment of the reals representing a timeline. Further, we will need to relativize the interpretation function over individuals, $\mathbf{I_D}$, to times:

- $\mathbf{I_D} : \mathbf{L} \times \mathbf{T} \to \mathcal{P}(\mathbf{D})$

where $\mathbf{L}$ is the set of terms in the language. We will also need a second extension function, $\mathbf{O_D}$, to represent the set of things that are F including those things which may not be actually existent:

- $\mathbf{O_D} : \mathbf{L} \times \mathbf{T} \to \mathcal{P}(\mathbf{D})$

We require that $\mathbf{I_D}(T,t) \subseteq \mathbf{O_D}(T,t)$ for all terms T and times t. $\mathbf{I_D}$ may now be thought of as a description of an "inner" domain that varies with time, making the logic generated by this model theory a free logic. Since we do not require that only actually existent individuals at t occur within the extension of the interpretation function $\mathbf{O_D}$, the logic generated by these models will be a free logic with positive semantics.[91]

We may now re-define *ut nunc* and *simpliciter* assertorics as follows, in line with the conclusions of the discussion above:

- AaB is true in $\mathcal{M}$ at t iff $\mathbf{I_D}(B,t) \neq \varnothing \wedge \mathbf{I_D}(B,t) \subseteq \mathbf{I_D}(A,t)$
- AeB is true in $\mathcal{M}$ at t iff $\mathbf{I_D}(B,t) \cap \mathbf{I_D}(A,t) = \varnothing$
- AiB is true in $\mathcal{M}$ at t iff AeB is not true in $\mathcal{M}$ at t
- AoB is true in $\mathcal{M}$ at t iff AaB is not true in $\mathcal{M}$ at t

- $AXaB$ is true in $\mathcal{M}$ at t iff $\mathbf{O_D}(B,t) \subseteq \mathbf{O_D}(A,t)$ is true in $\mathcal{M}$ for all t
- $AXeB$ is true in $\mathcal{M}$ at t iff $\mathbf{O_D}(B,t) \cap \mathbf{O_D}(A,t) = \varnothing$ is true in $\mathcal{M}$ for all t
- $AXiB$ is true in $\mathcal{M}$ at t iff $\mathbf{O_D}(B,t) \cap \mathbf{O_D}(A,t) \neq \varnothing$ is true in $\mathcal{M}$ for all t
- $AXoB$ is true in $\mathcal{M}$ at t iff $\mathbf{O_D}(B,t) \nsubseteq \mathbf{O_D}(A,t)$ is true in $\mathcal{M}$ for all t

Here the *ut nunc* assertorics are unchanged from Thom's except for being relativized to a given time, and restricted to the inner domain [33, p. 99]. *Simpliciter* assertorics differ from plain assertorics according to these definitions in three ways: First, they require their condition to hold not

[91] On the classification of free logics see Nolt [23, sect. 3.2].

just at one time but "perpetually"—that is, at all times.[92] Second, they concern also non-actual individuals at any given time. And third, the universal affirmatives do not have existential import.[93] In order to ensure the necessity and sufficiency of *simpliciter* predication for intensional relations, we may ascribe to Kilwardby the following *bridging principles* which are required to hold in any model:

1. $(\forall t \in \mathbf{T})(\mathbf{O_D}(A,t) \subseteq \mathbf{O_D}(B,t))$ if, and only if, $\mathbf{I_F}(A) \Leftarrow \mathbf{I_F}(B)$
2. $(\forall t \in \mathbf{T})(\mathbf{O_D}(A,t) \cap \mathbf{O_D}(B,t) = \varnothing)$ if, and only if, $\mathbf{I_F}(A) \Downarrow \mathbf{I_F}(B)$
3. $(\forall t \in \mathbf{T})(\mathbf{O_D}(A,t) \nsubseteq \mathbf{O_D}(B,t))$ if, and only if, $\mathbf{I_F}(A) \not\Leftarrow \mathbf{I_F}(B)$
4. $(\forall t \in \mathbf{T})(\mathbf{O_D}(A,t) \cap \mathbf{O_D}(B,t) \neq \varnothing)$ if, and only if $\mathbf{I_F}(A) \not\Downarrow \mathbf{I_F}(B)$

These principles may be compared with the "principle of plenitude":[94] They say, in effect, that there must be an intensional relation between terms underlying any eternal relation among extensions. However, they stipulate this equivalence with respect to extensions within the outer domain, not the inner domain.

Thom's remaining truth conditions can then be interpreted as they stand, with changes made only to relativize each truth condition to a given moment in time as with the *ut nunc* assertorics above. If we modify the definition of semantic entailment in the obvious way,[95] then all of Thom's positive results concerning *simpliciter* assertorics will remain intact, since, given the above bridging principles, these entail (and are entailed by) the intensional conditions taken to be their truth conditions by Thom.[96]

This distinction between an inner and an outer domain allows us to capture what makes a contingency natural for Kilwardby. What makes "men go grey" a distinctively natural kind of contingency, on Kilwardby's

[92] *perpetuum*, Robert Kilwardby, *Notulae super librum Perihermeneias*, pt. 1, L.2 (*ad* 16a18), quoted in Thom [33, 82n17].

[93] Cf. Thom [33, p. 100]. Thanks to Christophe Geudens for bringing my attention to issues of existential import here.

[94] See Hintikka [10]; for the prevalence of these assumptions in early scholastic philosophy, see Knuuttila [14] and Knuuttila [15], especially ch. 3.

[95] $\mathcal{M} \vDash_t \phi$ iff ϕ is true in $\mathcal{M}$ at t; $\mathcal{M} \vDash_t \{\phi_1, \ldots, \phi_n\}$ iff $\mathcal{M} \vDash_t \phi_1, \ldots, \mathcal{M} \vDash_t \phi_n$; and $\Gamma \vDash \phi$ iff for all models $\mathcal{M}$, for all times t, if $\mathcal{M} \vDash_t \Gamma$, then $\mathcal{M} \vDash_t \phi$.

[96] Some additional modifications may be needed to this definition in order to stop it from validating unwanted *invalidity* results. I will not consider this issue here.

analysis, is the satisfaction of the additional condition that any given man *would eventually go grey, were he to live a sufficiently long time*. In these models, we can capture this by the condition that every man is, at some point in the future when that man *may or may not exist*, part of the (outer) extension of "grey".[97] If you like: All men eventually do go grey, but this may occur after that man has ceased to exist.

That suggests the following truth-conditions for natural contingencies in these models:

- $\mathcal{M} \vDash AQ_{nat}aB$ iff (i) $M \vDash AQaB$, and (ii) $(\forall t \in \mathbf{T})(\forall x \in \mathbf{D})(x \in \mathbf{O_D}(B,t) \supset (\exists t' \geq t)(x \in \mathbf{O_D}(A,t')))$
- $\mathcal{M} \vDash AQ_{nat}iB$ iff (i) $M \vDash AQiB$ and (ii) $(\forall t \in \mathbf{T})(\exists x \in \mathbf{D})(x \in \mathbf{O_D}(B,t) \wedge (\exists t' \geq t)(x \in \mathbf{O_D}(A,t')))$

That is, a natural contingency requires a corresponding contingency to be true (where this is understood, as per Thom's analysis, as an intensional condition), and *also* for there to be a future time when the potentiality is realized (at which time the subject in question may or may not exist!). The condition is required to hold at all times so as to capture the fact that we are talking about *natural* processes: If it is *natural* for some or all men to go grey, we assume that this is a process that applies to men at all times (even if, as Kilwardby stresses, the process does not always reach completion). We define only positive statements, in line with Kilwardby's view that "natural contingencies [...] are not much mentioned under a negation but always positively" [35, pt. 1, pp. 578–588 (*dub.* 9)].[98]

No modifications will be required to Thom's truth conditions for other syllogistic modalities which, as mentioned, are formulated in terms of primitive conceptual relations over $\mathbf{I_F}$.[99] The only assumption we need to

[97] I am assuming here that a man exists if, and only if, that man is alive.

[98] I dispense here with Thom's requirements that the subject be an essence term and the predicate a denominative, since it is my goal to find a formalization that captures the logical results Kilwardby holds in connection with natural contingency, and these extra conditions are not required for any of these results as far as I can see. Nevertheless, it may be a part of Kilwardby's conception of natural contingency that the terms be restricted in this way, in which case these conditions could be added to the truth conditions above.

[99] I do not list these truth conditions, since they are not needed for the results

make about the logic of necessities is that negative universal necessities imply *simpliciter* (temporally unrestricted) assertorics defined here:

> If $\mathcal{M} \vDash_t ALeB$ then $\mathcal{M} \vDash_t AXeB$ (Necessity Implies Omnitemporality)

Kilwardby endorses this principle, although he denies the converse [35, pt. 1, p. 514.387–392].[100] Similar requirements for a, i and o propositions could be introduced but I will not do so here since they are also not needed for the results proven below.[101]

In order to simplify the representation of natural contingencies and to capture Kilwardby's other claims about this modality, we introduce two new term-level operators. Let $\dot{A}$ be read as "has reached the completion of a natural process which naturally terminates in A", and define $\tilde{A}$ so as to mean "is naturally undergoing a process which culminates in having A, or has reached the completion of this process." I will refer to a term of the form $\dot{A}$ as a result-term and one of the $\tilde{A}$ as a process-term. In order to capture the intended meaning of these operators in our models, we specify the following three axioms concerning result- and process-terms:

Axiom 1 $\mathbf{O_D}(\tilde{G}, t) \subseteq \mathbf{O_D}(\dot{G}, t')$ for some $t' \geq t$ [**natural processes reach completion**]

Axiom 2 $\mathbf{O_D}(\dot{G}, t) \subseteq \mathbf{O_D}(\dot{G}, t')$ for every $t' \geq t$ [**completion of processes cannot be undone**]

Axiom 3 $\mathbf{O_D}(\dot{G}, t) \subseteq \mathbf{O_D}(\tilde{G}, t')$ for every $t' \leq t$ [**all natural results are the result of a natural process**]

Axiom 1 says that, if there is a time at which x is in the extension of $\tilde{A}$, there is a posterior time at which it is in the extension of $\dot{A}$. For instance, if there is a time at which a man is going grey, there is a posterior time at which that man is grey. Note that axiom 1 does *not* specify that x still be an existing individual at t'. This is important, since it means that axiom 1 does not say (in the case of men going grey) that every

derived here. They are given in Thom [33, pp. 126–127, 153–154].

[100] This also holds on Thom's definition of *simpliciter* assertorics, which I have stipulated mine to be logically equivalent to. Hence this will hold on Thom's semantics, with the above modifications.

[101] These would also need to be modified because Kilwardby takes the subject terms of some modal propositions to be unampliated: See Thom [33, p. 120].

man who is in the process of going grey will live to actually be grey. The posterior time may be one at which the individual x is no longer in existence, in which case the axiom only requires that the individual *would be grey, were he still alive.*[102]

Axiom 2 specifies that the results of natural processes are permanent: If something gets grey hair as a result of this process, then it will always have naturally grey hair. Hair treatments may mask the natural colour of one's hair, but they do not change the natural colour of one's hair. In other words, the effect of having completed a natural process cannot be undone.

Finally, axiom 3 says that natural results do not come out of nowhere: Each individual with a natural result G (e.g. having grey hair) either already had G or was in the process of getting it at all prior times.[103]

It can then be shown that:

- $\mathcal{M} \vDash_t \dot{A}Q_{nat}aB$ iff (i) $M \vDash_t \dot{A}QaB$, and (ii) $M \vDash_t \tilde{A}XaB$
- $\mathcal{M} \vDash_t \dot{A}Q_{nat}iB$ iff (i) $M \vDash_t \dot{A}QiB$ and (ii) $M \vDash_t \tilde{A}XiB$

In other words, the completion of a natural process is predicated naturally contingently, if, and only if, the corresponding (plain) contingency holds and it is always the case that subjects of that sort naturally have that property or are in the process of receiving it. This is proven in the appendix (4.1).

Now, given the bridging principles stated above, we may infer from a temporally unrestricted assertoric that a primitive conceptual relation holds. It follows that clause (ii) may be rewritten to give the following truth conditions for natural contingencies:

- $\mathcal{M} \vDash_t \dot{A}Q_{nat}aB$ iff (i) $M \vDash_t \dot{A}QaB$, and (ii) $\mathbf{I_F}(\tilde{A}) \Leftarrow \mathbf{I_F}(B)$
- $\mathcal{M} \vDash_t \dot{A}Q_{nat}iB$ iff (i) $M \vDash_t \dot{A}QiB$ and (ii) $\mathbf{I_F}(\tilde{A}) \not\Leftarrow \mathbf{I_F}(B)$

[102] Readers who are comfortable with possibilia might prefer to say that the now-merely-possible-man turns grey.

[103] Here, for simplicity's sake, I make the admittedly artificial assumption that natural processes extend back infinitely in time, so that, for example, each possible man was already going grey before that man was born.

For a result-term, a natural contingency is therefore equivalent to the holding of a regular contingency [35, pt. 1, pp. 133–134] where there is also a necessary intensional relationship between the subject concept and the concept of the predicate's process or completion. This result captures Kilwardby's view of the relationship between natural contingency and necessity, according to which a natural contingency is closely associated with a necessary predication without being equivalent to one [35, pt. 1, pp. 535–536].[104]

It is also easy to see that, on these semantics, the version of complementary conversion Kilwardby states for natural contingencies does in fact hold. This is proven in appendix (4.2). Further, under the plausible assumption that true negative *per se* necessities are true at all times, it can be shown that these agree with Kilwardby's syllogistic results for natural contingencies. The central result, as we have seen, is Celarent $\mathrm{LQ_{nat}X_{simp}}$, where the conclusion is to be understood as temporally unrestricted.

We can also show that Celarent $\mathrm{LQ_{nat}X_{simp}}$ holds, so long as we consider predicates that describe the completion of natural processes, as Kilwardby does.[105] Suppose that in some model $\mathcal{M}$, for some time t, (i) $\mathcal{M} \vDash_t \dot{O}Le\dot{G}$ and (ii) $\mathcal{M} \vDash_t \dot{G}Q_{nat}aM$. For vividness, read O as "has orange hair", G as "has grey hair" and M as "is a man"; then these say that necessarily nothing which has naturally obtained orange hair has naturally obtained grey hair and it is naturally contingent for all men to naturally obtain grey hair. Now, if it is necessary for nothing naturally grey to be naturally orange, and if all men naturally contingently have grey hair, then no

[104] The relevant necessity will not be the *per se* type of necessity Kilwardby takes to be at issue in the modal syllogistic, since it neither expresses a genus-species relation in the category of substance nor a relationship between two abstract terms (like "white is coloured"); it will, however, plausibly be what Kilwardby elsewhere calls a necessity *per accidens*: See Thom [33, pp. 113–119].

[105] I will not attempt to show that the mood is unrestrictedly valid. Kilwardby's theory of natural contingency only motivates considering the result for predicate terms which represent the completion of natural processes, since this is the reading of the predicate required in order to make a natural contingency true. I therefore restrict the minor premise to a result-term. I make the further assumption that the other term in the necessity is also a result-term. Note, however, that the $\tilde{}$ and $\dot{}$ operators allow us to express this as a formally valid argument.

man ever has naturally orange hair, since there would then need to be a future time at which the possible or actual man still has orange hair (since natural processes cannot be undone; axiom 2) but at which time he also has naturally grey hair (since natural processes reach completion; axiom 1). A more formal proof is given in the appendix (4.4), as well as an analogous proof for Ferio $\mathrm{LQ}_{\mathrm{nat}}\mathrm{X}_{\mathrm{simp}}$. Since the third syllogistic mood involving natural contingency that Kilwardby recognizes, Cesare $\mathrm{LQ}_{\mathrm{nat}}\mathrm{X}_{\mathrm{simp}}$, reduces to Celarent by conversion, this suffices to capture Kilwardby's syllogistic logic of natural contingency.

3.3 Concluding remarks: Natural contingency in Kilwardby and beyond

While less developed than his theory of *per se* necessity and possibility, Kilwardby's remarks about natural contingency provide enough detail to give the outlines of an interesting logical system. It is probably still correct to describe Kilwardby's logic as an intensional system, but the foregoing shows that there is at least one type of modality where extensional and temporal notions are important to the way Kilwardby understands the meaning of modal language.

I am not aware of explorations of the logic of natural processes and results in contemporary logic.[106] A project for future research would be to consider the logic of a language of temporal predicate logic that included process- and result-forming operators for predicates, and explore the modalities that naturally arise in it. If we abstract from Kilwardby's reliance on a syllogistic framework and consider only his treatment of natural contingency as a modality, then a natural first step in formalizing a modal operator of natural contingency along the lines of Kilwardby's analysis in a predicate-logical language might be to define, for atomic sentences:

$$\Diamond_{nat}G(a,t) \equiv_{def} \Diamond G(a,t) \wedge \Diamond \neg G(a,t) \wedge (\exists t')(\forall t'' \geq t')(E!(a,t'') \rightarrow G(a,t''))$$

[106] The logic of *processes* has been explored [see e.g. 25, pp. 155–169], but I know of no explorations of the logic of processes that are distinctively *natural* and their natural outcomes. For a formal analysis of the metaphysics of some closely related notions, however, see Freddoso [9].

That is, it is naturally contingent that Ga at some given time t if, and only if, it is at that time possible that Ga and possible that $\neg Ga$ and there is a future time after which a will be G, so long as a exists (here $E!$ is an existence predicate). I leave it to future research to consider how, or whether, this could be generalized beyond the case of atomic propositions.

4 Appendix: Supplementary technical results

4.1 Natural contingency truth conditions with process- and result-terms

We show that given the general definition of natural contingency

> $\mathcal{M} \vDash_t AQ_{nat}aB$ iff (i) $M \vDash_t AQaB$, and (ii) $(\forall x \in \mathbf{D})(\forall t \in \mathbf{T})(x \in \mathbf{O_D}(B,t) \supset (\exists t' \geq t)(x \in \mathbf{O_D}(A,t')))$

it follows that

> $\mathcal{M} \vDash_t \dot{A}Q_{nat}aB$ iff (i) $M \vDash_t \dot{A}QaB$, and (ii) $M \vDash_t \tilde{A}XaB$

The first clause of the definiens obviously agrees with the first clause of the general definition. It needs to be shown that the second clause agrees with the general definition, i.e. that $(\forall x \in \mathbf{D})(\forall t \in \mathbf{T})(x \in \mathbf{O_D}(B,t) \supset (\exists t' \geq t)(x \in \mathbf{O_D}(\dot{A},t'))) \Leftrightarrow M \vDash_t \tilde{A}XaB$. We argue as follows:

Right to left: Suppose that $M \vDash_t \tilde{A}XaB$. Then $\mathbf{O_D}(B,t') \subseteq \mathbf{O_D}(\tilde{A},t')$ for all t' by the *simpliciter* assertoric truth condition. By axiom 1 there is a $t'' \geq t'$ such that $\mathbf{O_D}(B,t') \subseteq \mathbf{O_D}(\dot{A},t'')$, for any t'. That is to say, $(\forall x \in \mathbf{D})(\forall t \in \mathbf{T})(x \in \mathbf{O_D}(B,t) \supset (\exists t' \geq t)(x \in \mathbf{O_D}(\dot{A},t')))$

Left to right: Suppose that $(\forall x \in \mathbf{D})(\forall t \in \mathbf{T})(x \in \mathbf{O_D}(B,t) \supset (\exists t' \geq t)(x \in \mathbf{O_D}(\dot{A},t')))$. Take any given time r. If there is no $a \in \mathbf{O_D}(B,r)$, then the conditional is trivially satisfied. If there is such an a, then $a \in \mathbf{O_D}(\dot{A},s)$ for some time $s \geq r$. Now by axiom 3, $a \in \mathbf{O_D}(\tilde{A},r)$. Since a is an arbitrary member of $\mathbf{O_D}(B,r)$, it follows that $\mathbf{O_D}(B,r) \subseteq \mathbf{O_D}(\tilde{A},r)$. Since r is an arbitrary time it follows that, for all times t', $\mathbf{O_D}(B,t') \subseteq \mathbf{O_D}(\tilde{A},t')$. This is the truth condition for $\tilde{A}XaB$. Hence, $M \vDash_t \tilde{A}XaB$.

The proof for the particular affirmative is analogous.

4.2 Q_{nat}/M conversion

These follow trivially given the rules of (i) complementary conversion ($AQaB \dashv\vDash AQeB$ and $AQiB \dashv\vDash AQoB$) (ii) Q-M weakening ($AQ \circ B \vDash AM \circ B$ for $\circ = a, e, i, o$)

1. $AQ_{nat}aB \vDash AMeB$. Proof: Suppose $AQ_{nat}aB$ is true in a model $\mathcal{M}$. By the truth condition for $AQ_{nat}aB$, $AQaB$ holds in $\mathcal{M}$. By complementary conversion, $AQeB$ holds in $\mathcal{M}$. Then by Q-M weakening, $AMeB$ holds in $\mathcal{M}$.

2. $AQ_{nat}iB \vDash AMoB$. Proof: Suppose $AQ_{nat}iB$ is true in a model $\mathcal{M}$. By the truth condition for $AQ_{nat}iB$, $AQiB$ holds in $\mathcal{M}$. By complementary conversion, $AQoB$ holds in $\mathcal{M}$. Then by Q-M weakening, $AMoB$ holds in $\mathcal{M}$.

4.3 Ferio $\mathbf{LQ_{nat}X_{simp}}$

We will show that $\dot{O}Le\dot{G}, \dot{G}Q_{nat}iM \vDash \dot{O}XoM$.

Suppose that in some model $\mathcal{M}$, for some time t, (i) $\mathcal{M} \vDash_t \dot{O}Le\dot{G}$ and (ii) $\mathcal{M} \vDash_t \dot{G}Q_{nat}iM$. Assume for reductio that the temporally unrestricted conclusion were false at t. Then there is a time s such that $\mathbf{O_D}(M, s) \subseteq \mathbf{O_D}(\dot{O}, s)$. By (ii), $\mathbf{O_D}(M, s) \subseteq \mathbf{O_D}(\tilde{G}, s)$. Let $a \in \mathbf{O_D}(M, s)$ (we are guaranteed such an individual by (ii)). Then $a \in \mathbf{O_D}(\dot{O}, s)$ and $a \in \mathbf{O_D}(\tilde{G}, s)$. By axioms 1 and 2, however, there is a time $v \geq s$ such that $a \in \mathbf{O_D}(\dot{G}, s)$ and $a \in \mathbf{O_D}(\dot{O}, s)$. Since Necessity Implies Omnitemporality, however, $\mathbf{O_D}(\dot{G}, s) \cap \mathbf{O_D}(\dot{O}, s) = \varnothing$, a contradiction. We therefore reject the assumption for reductio and conclude that a simpliciter assertoric conclusion holds. That is, Ferio $\mathrm{LQ_{nat}X_{simp}}$ is valid for completion predicates.

4.4 Celarent $\mathbf{LQ_{nat}X_{simp}}$

We will show that $\dot{O}Le\dot{G}, \dot{G}Q_{nat}aM \vDash \dot{O}XeM$.

Suppose that in some model $\mathcal{M}$, for some time t, (i) $\mathcal{M} \vDash_t \dot{O}Le\dot{G}$ and (ii) $\mathcal{M} \vDash_t \dot{G}Q_{nat}aM$. Assume for reductio that the temporally unrestricted

conclusion were false. Then by the truth condition for a universal negative *simpliciter* assertoric, there is a time u and an individual a for which $a \in \mathbf{O_D}(\dot{O}, u)$ and $a \in \mathbf{O_D}(M, u)$. By (ii), however, $a \in \mathbf{O_D}(\tilde{G}, u)$, and so by axiom 1 there is a future time v such that $a \in \mathbf{O_D}(\dot{G}, v)$. At this time, however, we also have $a \in \mathbf{O_D}(\dot{O}, v)$ by axiom 2. By Necessity Implies Omnitemporality, however, (i) implies that $\mathbf{O_D}(\dot{G}, v) \cap \mathbf{O_D}(\dot{O}, v) = \varnothing$, a contradiction. We therefore reject the assumption for reductio and conclude that a simpliciter assertoric conclusion holds. That is, Celarent $\mathrm{LQ}_{\mathrm{nat}}\mathrm{X}_{\mathrm{simp}}$ is valid for completion predicates.

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