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Term Kinds and the Formality of Aristotelian Modal Logic

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Recent formalizations of Aristotle's modal syllogistic have made use of an interpretative assumption with precedent in traditional commentary: That Aristotle implicitly relies on a distinction between two classes of terms. I argue that the way Rini (2011. *Aristotle's Modal Proofs: Prior Analytics A8–22 in Predicate Logic*, Dordrecht: Springer) employs this distinction undermines her attempt to show that Aristotle gives valid proofs of his modal syllogisms. Rini does not establish that Aristotle gives valid proofs of the arguments which she takes to best represent Aristotle's modal syllogisms, nor that Aristotle's modal syllogisms are instances of any other system of schemata that could be used to define an alternative notion of validity. On the other hand, I argue, Robert Kilwardby's ca. 1240 commentary on the *Prior Analytics* makes use of a term-kind distinction so as to provide truth conditions for Aristotle's necessity propositions which render Aristotle's conversion rules and first figure modal syllogisms formally valid. I reconstruct a *suppositio* semantics for syllogistic necessity propositions based on Kilwardby's text, and yield a consequence relation which validates key results in the assertoric, pure necessity and mixed necessity-assertoric syllogistics.

1. Introduction

Over the last three decades, significant advances have been made in understanding chapters 3 and 8–22 of *Prior Analytics* I, where Aristotle extends his syllogistic to modal propositions. Many of the most successful recent interpretations have drawn on Aristotle's remarks about necessity and contingency outside the *Prior Analytics* in order to interpret Aristotle's text. Careful analyses of passages from Aristotle's *Topics*, *Categories* and *Posterior Analytics* have paid off in interpretations which attribute fewer or more subtle logical mistakes to Aristotle.¹ In this way, modern interpretations have come to bear certain resemblances to those of the early scholastics, such as Robert Kilwardby's, whose ca. 1240 commentary on the *Prior Analytics* develops a semantic interpretation of Aristotle's modal logic by situating it in the context of Aristotle's corpus.²

The study of Aristotle's modal syllogistic due to Rini 2011 bears a complicated relationship to this programme of interpretation. Her primary goal differs from that of many other modern interpretations of Aristotle's modal logic. Whereas Nortmann 2002, 1996, Thomason 1993, 1997 and McCall 1963 all aim to provide formal systems and semantic interpretations which capture exactly the modal syllogisms Aristotle endorses, Rini concentrates on finding an adequate representation of Aristotle's proofs of these modal syllogisms, using only logical resources that Aristotle himself could have plausibly had in mind (Rini 2011, p. 1).³ She believes that the key to successfully representing these proofs is to postulate that Aristotle recognizes a class of terms which belong of necessity to all

¹ Among others, van Rijen 1989, Patterson 1995, Nortmann 1996, Thom 1996, Malink 2006, 2013. For a more sceptical appraisal of these approaches, see Smith 1989 (p. xxxvi) and Striker 2009 (pp. xvi–xvii).

² Thom 2007 (p. 11). See Lagerlund 2008 on the assimilation of Aristotle's works into logical tradition of the Latin West.

³ An exception is Thom 1996, who gives a detailed treatment of Aristotle's proof methods alongside the formal systems which he develops.

that they belong to, and necessarily do not belong to all that they do not belong to. That is, *Rini 2011* (p. 3, 5) claims that Aristotle takes there to be predicates ϕ for which the following two conditions hold:

$$\forall x(\phi x \equiv L\phi x) \quad (\text{Substance Principle})$$

and

$$\forall x(\neg\phi x \equiv L\neg\phi x) \quad (\text{Negative Substance Principle})$$

Perhaps with the aim of sidestepping philological issues regarding the role and origin of this principle, *Rini 2011* (p. 41) chooses a neutral vocabulary for these terms, calling those terms which obey the principle ‘red’, and those which do not obey this principle ‘green’.⁴ Rini holds that this distinction is all that is needed to successfully represent Aristotle’s modal proofs in first-order logic, and she employs it to treat many traditional problems of interpretation.

For instance, the problem of the ‘Two Barbaras’ asks to explain how Aristotle can maintain that Barbara LXL is valid but Barbara XLL invalid (*Prior Analytics* I.9, 30a17–19, 30a23–25).⁵ Most modern interpreters have taken Aristotle’s further claim that ‘A is said necessarily of all B’ entails ‘B is said necessarily of some A’ to rule out a simple *de re* reading of affirmative necessities, and have made use of more complex constructions.⁶ Rini, however, suggests retaining a *de re* reading of modal propositions, representing Aristotle’s ‘A is said necessarily of all B’ in first-order logic as $\forall x(Bx \supset LAx)$, and giving a corresponding Frege-Russell analysis of particular and negative propositions with all necessities interpreted as *de re* modifiers of the predicate expression (*Rini 2011*, p. 52).

The choice to maintain a simple *de re* analysis of syllogistic necessities reflects Rini’s programme of showing that interpreters have tended to overcomplicate the task of interpreting Aristotle’s logic. Rini contends that complex formalisms and exotic logics become unnecessary for giving an account of Aristotle’s modal proofs as soon as we take on board the distinction between ‘red’ and ‘green’ terms (*Rini 2011*, pp. 42–44, 59). Her book aims to show that, given this distinction, a small fragment of modal predicate logic is adequate for formalizing Aristotle’s modal syllogisms as well as his proofs of these syllogisms.⁷

⁴ *Rini 2011* (pp. 4, 39–40) bases this principle loosely on the *Posterior Analytics*, but unlike *Malink 2013*, she does not make many specific claims regarding the relationship of logical principles needed for the interpretation of syllogistic to Aristotle’s broader philosophy.

⁵ Here and throughout, I adopt notational conventions that Rini derives from *McCall 1963*: I refer to syllogisms by their standard medieval mnemonics (viz. Barbara, Celarent, etc.) and three letters to indicate whether the major premise, minor premise and conclusion are respectively a necessity (L) or an assertoric (X) proposition. So, Barbara LXL refers to the argument from ‘A is said necessarily of all B’ and ‘B is said of all C’ to ‘A is said necessarily of all C’. I will also use the standard medieval conventions for referring to the quality and quantity of propositions (‘a’ for universal affirmative, ‘i’ for particular affirmative, ‘e’ for universal negative and ‘o’ for particular negative). For further details, see *Lagerlund 2015*.

⁶ *McCall 1963* (pp. 18–21) rejects the attempt of *Becker 1933* to interpret Aristotle’s necessities as *de re* modals on the grounds that this leads Becker to conclude that Aristotle’s logic is plagued with equivocations. *Malink 2006, 2013* and *Nortmann 1996* employ more complex constructions in first-order logic which validate more of Aristotle’s results. *Thomason 1997* and *Johnson 1989* give *de re* readings of Aristotle’s modal syllogistic, but provide their own semantics for such propositions. *Thom 1996* provides a number of semantic interpretations for a formal language developed for the purpose of representing Aristotle’s syllogistic.

⁷ Specifically, Rini formalizes Aristotle’s modal syllogisms and proofs in a small fragment of modal predicate logic consisting of sentences of the form $\exists x(Fx \wedge \bullet \star Gx)$ and $\forall x(Fx \supset \bullet \star Gx)$ where \bullet is replaced with a negation sign or deleted, and \star is replaced with one of Rini’s three modal sentential operators (L – necessity, M – possibility, and Q – contingency) or deleted. No special assumptions are made about the logic of L, M and Q except that L satisfies the T-axiom ($L\phi \supset \phi$) and that L and M satisfy the dual axioms.

Accordingly, Rini represents Barbara LXL as the following argument in modal predicate logic (*Rini 2011*, p. 74):

$$\frac{\forall x(Bx \supset LAx) \quad \forall x(Cx \supset Bx)}{\forall x(Cx \supset LAx)} \quad (1)$$

This argument is straightforwardly valid, and does not depend on any special principles of modal logic. In the same way it can be seen that Rini's formalization of Barbara XXL is invalid, as Aristotle claims (30a23–25). The challenge is to explain how Aristotle can consistently maintain this while simultaneously holding that 'A is said necessarily of all B' converts to 'B is said necessarily of some A', since from

$$\forall x(Bx \supset LAx)$$

it does not, on any obvious logic for L , follow that

$$\exists x(Ax \wedge LBx).$$

Here Rini invokes her 'Substance Principle' and claims that this conversion will indeed hold so long as B is a 'red' term, so that $Bx \equiv LBx$ holds.⁸ Similar arguments are made for the conversion of universal negative propositions.⁹

Rini concludes that necessity conversion rules can only be applied when the subject term of the proposition is 'red', a condition she calls the 'Genuineness Requirement' (*Rini 2011*, p. 4). For instance, *Rini 2011* (p. 79) represents Cesare LXL as the following argument in modal predicate logic:

$$\frac{\forall x(Bx \supset L\neg Ax) \quad \forall x(Cx \supset Ax)}{\forall x(Cx \supset L\neg Bx)} \quad (2)$$

Rini's way of representing Aristotle's proof of Cesare LXL from Celarent LXL, when made fully explicit, comes to the following (*Rini 2011*, p. 80):

- 1 $\forall x(Bx \supset L\neg Ax)$ (premise)
- 2 $\forall x(Cx \supset Ax)$ (premise)
- 3 $\forall x(Bx \supset \neg Ax)$ (from 1 using $L\varphi \Rightarrow \varphi$)
- 4 $\forall x(Ax \supset \neg Bx)$ (from 3 using contraposition and DNE) (3)
- 5 $\forall x(Ax \supset L\neg Bx)$ (from 4 by the Negative Substance Principle,
assuming B is a 'red' term)
- 6 $\forall x(Cx \supset L\neg Bx)$ (from 2 and 5 by Celarent LXL)

⁸ *Rini 2011* (pp. 72–78). The assumption that all terms are non-empty is also required to validate this inference. *Rini 2011* (p. 28) claims to secure this assumption by stipulating that Aristotle's syllogistic logic was intended only to be applied to non-empty terms, in the same way that the modal syllogistic requires certain terms to be 'red'. Questions could be raised regarding the legitimacy of this stipulation similar to those which I raise regarding the legitimacy of term restrictions below. I will not, however, dwell on issues of existential import in this paper.

⁹ *Rini 2011* (p. 54).

The validity of lines 1–4 and 6 is not problematic.¹⁰ Line 5 however requires an assumption not derivable from premises 1 and 2: That B is a ‘red’ term. Consequently, the argument in (3) is not formally valid, nor can any formally valid proof of (2) be given, within the constraints of Rini’s interpretative paradigm.¹¹ Hence, Cesare LXL itself is also not formally valid on Rini’s account. It only preserves truth when B is a ‘red’ term.

The same considerations hold for other mixed-mode syllogisms. Since Rini’s conversion rules only apply to ‘red’ terms, any use of a conversion rule in Aristotle’s proofs of modal syllogisms will require the unstated assumption that the predicate term be ‘red’. Consequently, on Rini’s formalizations, Aristotle does not prove his mixed-mode necessity syllogisms using arguments that are valid for arbitrary substitutions of terms. As a result, many of Aristotle’s modal proofs are not formally valid on Rini’s interpretation.

Nevertheless, *Rini 2011* (p. 80) claims that this does not mean we should say that Aristotle’s arguments which require term restrictions are invalid. Rather, as Rini puts it, ‘validity in these cases means validity for appropriately restricted terms’ (*Rini 2011*, p. 80). According to Rini, even though Aristotle’s modal syllogisms are not good arguments in the sense that they preserve truth for all choices of A , B , and C , they are nonetheless ‘valid’ when the terms substituted allow the Substance Principle and the Negative Substance Principle to be employed as needed.

Rini defends an unusual position. One would expect that a formalization intended to show a body of arguments to be valid would do so by providing a formalization on which these arguments are formally valid. Rini however claims that her formalization of Aristotle’s proofs is adequate, and that the proofs thus formalized are valid, but denies that Aristotle’s proofs are *formally* valid.

Rini 2011 does not offer a clear statement of what this means. In one passage, Rini suggests that Aristotle’s syllogisms in the necessity syllogistic do not yield modal conclusions at all but merely non-modal propositions, the necessity of which is a ‘non-formal issue’ (*Rini 2011*, p. 26). On this interpretation, the conclusion of Cesare LXL would be the assertoric proposition $\forall x(Cx \supset \neg Bx)$. If B furthermore satisfies the Negative Substance Principle, someone could then go on to infer $\forall x(Cx \supset L\neg Bx)$. Cesare LXL would then be an argument in two steps:

$$\frac{\forall x(Bx \supset L\neg Ax)}{\forall x(Cx \supset Ax)} \quad (4a) \qquad \frac{\forall x(Cx \supset \neg Bx)}{\forall x(\neg Bx \equiv L\neg Bx)} \quad (4b)$$

$$\frac{\forall x(Cx \supset Ax)}{\forall x(Cx \supset \neg Bx)} \qquad \frac{\forall x(\neg Bx \equiv L\neg Bx)}{\forall x(Cx \supset L\neg Bx)}$$

(4a), which we can call the main argument, is trivially valid in modal predicate logic. (4b), which we can call the auxiliary step, is an inference to a modal conclusion from the assertoric conclusion of (4a) based on the assumption that B is a ‘red’ term, an assumption expressed in the premise $\forall x(\neg Bx \equiv L\neg Bx)$.

Construing Cesare LXL as an argument in two steps might seem to resolve the tension between the claims that Aristotle’s proofs can be formalized as valid arguments using first-order logic with the claim that these formalized arguments are not formally valid. On this construal, both steps (4a) and (4b) are formally valid arguments in first-order logic. Nevertheless, the considerations which allow a modal conclusion to be correctly inferred – namely, that B is a ‘red’ term, and so licenses the addition of an auxiliary premise

¹⁰ Lines 1–4 require only first-order logic and the T-axiom $L\phi \Rightarrow \phi$. ‘Celarent LXL’ in line 6 refers to the argument $\forall x(Bx \supset L\neg Ax), \forall x(Cx \supset Bx) \vdash \forall x(Cx \supset L\neg Ax)$, which is a substitution instance of the first-order valid schema $\forall x(\phi[x] \supset \psi[x]), \forall x(\chi[x] \supset \phi[x]) \vdash \forall x(\chi[x] \supset \psi[x])$.

¹¹ Here we should keep in mind that Rini explicitly disavows using any logical principles which Aristotle could not plausibly have had in mind (*Rini 2011*, p. 1), so invoking a bespoke logic for L is ruled out by Rini’s interpretative goals.

in (4b) – remain ‘ultimately semantic’ (Rini 1998, p. 555). The ‘restriction’ on the validity of Cesare LXL would then consist in the auxiliary step (4b) only being sound when the B term of the main argument (4a) is red.

However, this does not comport with Rini’s representation of Cesare LXL in (2), nor her representation of Aristotle’s proof of it in (3), which are each single arguments.¹² Furthermore, this interpretation makes the modality of the major premise of Celarent LXL otiose: Given that B is a ‘red’ term, the conclusion of (4a) can be ‘upgraded’¹³ to a necessity via (4b) regardless of whether either premise of the main argument is a necessity. To extend this interpretation to the entire necessity syllogistic would be to admit a serious interpretative cost. All of Aristotle’s arguments for the (in)validity of various combinations of necessity and assertoric premises would have been for nothing, since the syllogisms themselves only ever yield assertoric conclusions, regardless of which premises, if any, are necessity propositions. Modal conclusions would only ever be inferred from logically independent facts about which terms are ‘red’.¹⁴

As a result, it would be a confusion to even speak of valid or invalid modal *syllogisms*, since the cases where a necessity conclusion can be inferred do not depend on which premises of the syllogism are necessity propositions, but rather only on the kinds of terms that occur in them.¹⁵ There would be no non-trivial deductions from modalized premises to a modalized conclusion,¹⁶ and the modal ‘syllogistic’ would have to be counted as a conceptual confusion.¹⁷

It is therefore urgent that a different definition of ‘restricted validity’ be available to Rini if she is to have good grounds for maintaining that Aristotle’s modal syllogisms or their proofs are in any sense valid. In the following section, I consider some possibilities, but argue that Rini’s method of restricting terms so as to ensure the applicability of the Substance Principle and the Negative Substance Principle cannot be used to define a notion of validity for Aristotle’s modal syllogisms. By contrast, I will argue that Robert Kilwardby, although he employs a distinction between term-kinds parallel to Rini’s, succeeds in establishing that Aristotle’s mixed-mood necessity syllogisms are formally valid.

¹² In an earlier paper, Rini 1998 (p. 562) does not claim that the result which Aristotle succeeds in demonstrating with his proof of Cesare LXL is (2), but rather the following:

$$[\forall x(Bx \supset L\neg Ax) \wedge \forall x(\neg Bx \supset L\neg Bx) \wedge \forall x(Cx \supset Ax)] \supset \forall x(Cx \supset L\neg Bx). \quad (5)$$

This suggests that the argument which Aristotle has shown to be valid is therefore not (2) but rather:

$$\forall x(Bx \supset L\neg Ax), \forall x(\neg Bx \supset L\neg Bx), \forall x(Cx \supset Ax) \vdash \forall x(Cx \supset L\neg Bx). \quad (6)$$

However, (6) is plainly not the argument which Rini 2011 uses to represent Cesare LXL. The extra premise $\forall x(\neg Bx \supset L\neg Bx)$ has been added, and this premise is needed to yield the conclusion. An argument from $A \cup \{\phi\}$ to ψ does not show that $A \vdash \psi$. If Rini holds that Aristotle intended to prove (2) but slid into proving (6) instead, she gives us no reason to believe that Aristotle produced a proof of Cesare LXL that was valid in any sense.

¹³ For the terminology of ‘upgrading’, see McCall 1963 (p. 24), who uses it to describe the interpretation of Rescher 1963.

¹⁴ It can be verified that the semantic fact that A or B satisfies the (Negative) Substance Principle is logically independent from $Aa_L B$, $Ai_L B$, $Ae_L B$ or $Ao_L B$ on Rini’s formalizations.

¹⁵ Cf. Barnes 2012b. See also Kneale and Kneale 1962 (p. 91).

¹⁶ I say ‘non-trivial’ here since we would, of course, still validate those modal syllogisms whose formalizations are substitution instances of their assertoric counterparts, but these are not the cases generally considered to be problematic.

¹⁷ Barnes 2007 (p. 487) makes a similar point, and concludes that if ‘matter’ determines the modal status of the conclusion then there are no specifically modal syllogisms. On these grounds he draws the conclusion that the ‘modal arguments, pace Aristotle, [...] are not syllogisms; and they ought to be excluded from the purlieu of logic’. Rini 2011 (pp. 63–72) considers and explicitly rejects the position that the modal syllogistic is trivial in that there are no special syllogistic forms beyond those of the assertoric syllogistic.

2. Schematic and semantic concepts of formality

In order to explore what else might be made of Rini's claim that the arguments of the modal syllogistic are valid, but not formally valid, it is useful to draw on a recent taxonomy of senses in which logic is said to be formal due to *Dutilh-Novaes 2011*. Drawing on the work of *Etchemendy 1999* and *MacFarlane 2000*, *Dutilh-Novaes 2011* (pp. 307–314) distinguishes two broadly substitutional notions of formality. In the first sense, an argument is formally valid when it preserves truth regardless of which objects are referred to by referential expressions in the argument (*Dutilh-Novaes 2011*, pp. 310–314). However, this notion of formal validity is predated by a schematic conception articulated already in the Peripatetic School.¹⁸ In this latter sense, an argument is formally valid when it continues to preserve truth under arbitrary substitution of non-fixed vocabulary by symbols from the same syntactic class.¹⁹

Rini 2011 (pp. 41–42) is committed to denying that Aristotle's proofs are formally valid in the first sense, viz., that they preserve truth regardless of which entities terms refer to, since she claims that it is the sensitivity to the difference between terms which refer to what 'cannot be otherwise' and those which refer to what 'can be otherwise' that is responsible for the characteristic sort of validity which arguments in the mixed necessity syllogistic possess. She also appears to deny that Aristotle's arguments are formally valid in the second sense, since she takes them not to preserve truth for all predicate expressions; only 'red' predicates may be substituted in certain positions. While *Rini 2011* (pp. 63–64) admits that an interpretation which requires *all* terms to be 'red' has some textual support, her preferred interpretation takes Aristotle to allow both 'red' and 'green' terms in the mixed necessity syllogistic, subject to restrictions on terms so as to allow conversion (*Rini 2011*, p. 71).

Nevertheless, the concept of a schema is useful for making precise Rini's position. To this end, let us introduce some terminology for speaking about schematic validity. It is well known that there is no straightforward way to demarcate fixed from non-fixed parts of the logical vocabulary.²⁰ We shall therefore take schematic validity to be relative to a given *side-condition* which divides the vocabulary into a set of constant expressions \mathfrak{F} and a set of non-constant expressions \mathfrak{V} , and gives a specification of the range of expressions which may be substituted for each element of \mathfrak{V} . An *instance* of a schema can then be defined relative to a side-condition C as the result of substituting all schematic letters in the argument for elements from their ranges as given by the side-condition.²¹ An argument can then be defined to be valid subject to side-condition C (or ' C -valid') if all of its instances relative to C are truth-preserving.

Rini does in fact show the proof of Cesare LXL, as she represents it (cf. (3)) to be valid subject to the following side-condition:

$$\begin{aligned}
 \mathfrak{F} &= \{\forall, \neg, \supset, x, L, (,)\} \\
 \mathfrak{V} &= \{A, B, C\} \\
 \text{Ran}(A) &= \text{Ran}(C) = \{\text{Predicates in the language}\} \\
 \text{Ran}(B) &= \{\text{Red predicates in the language}\}.
 \end{aligned}
 \tag{7}$$

¹⁸ See Alexander of Aphrodisias *1991* (pp. 6, 16–21), whom *Dutilh-Novaes 2011* (p. 307) identifies as an early proponent of the schematic conception of formality.

¹⁹ This distinction stems from *Etchemendy 1999* (p. 28).

²⁰ See *Etchemendy 1999*, *MacFarlane 2000*.

²¹ See *Corcoran 2014*, who proceeds slightly differently, taking a schema to consist of a template text together with a side-condition rather than relativizing the notion of an instance.

This is a straightforward consequence of the way Rini represents Aristotle's proof of Cesare in (3): The argument she gives preserves truth over arbitrary predicate substitutions for A and C so long as B ranges only over 'red' terms (with the logical constants interpreted as part of the fixed vocabulary as usual).

Does this establish that there is a sense of 'valid' in which Aristotle's proofs are valid, after all? There are two arguments that might be given for why it does not. The first concerns the sorts of ranges used in (7). Intuitively, ranges serve to ensure that only terms of the right type are substituted for the schematic letters. The provision that only terms of the 'right type' be substituted is not, however, supposed to rule out substitution instances which would render the argument invalid, but rather substitutions which would yield non-well-formed formulas (for instance, the substitution of a singular term for a predicate). Therefore, the ranges of non-constant expressions given by a side-condition are usually required to be given by *syntactic* descriptions.²² Rini glosses the 'red'/'green' distinction in semantic terms,²³ and seems to treat being a 'red' term as a semantic notion. Whether this is her preferred way of thinking of the distinction or not, she is committed to it being a semantic distinction given her claim that first-order logic is adequate to formalize Aristotle's modal proofs, since first-order logic does not contain a syntactic class of predicate letters which always apply to their subjects necessarily. It can therefore be objected that (7) is not a legitimate side-condition, because it assigns as the range of one of its schematic letters a class of terms which is defined by a semantic, rather than a syntactic condition ('{Red predicates in the language}').

On the other hand, someone sympathetic to the term-restriction approach might reply that whether semantic classes are admitted into the side-condition is precisely what distinguishes formal from restricted validity. The substitution instances of (7), the reply goes, only preserve truth for a certain, semantically-defined class of terms (i.e. whenever the substituent of B is one which satisfies the (Negative) Substance Principle), and this is just what it means to say that the argument is restrictedly valid. This suggests the following definitions. A side-condition C is *restrictive* if the range of at least one schematic letter is defined by a semantic, rather than a syntactic, condition.²⁴ An argument T is *formally valid* subject to side-condition C if it is C -valid and C is not restrictive; it is *restrictedly valid* subject to C if it is C -valid and C is restrictive.

Thus defined, there is not a single sense of 'restricted validity', but rather there is one for every restrictive side-condition. This relativization is necessary for the definition to avoid triviality. If instead we define an argument to be restrictedly valid (period) whenever it is an instance of *any* restrictedly valid schema, then the definition would include arguments which are intuitively invalid, since it is in general trivial to gerrymander a side-condition which will validate a given argument. For instance, the argument form ' X is a dog, therefore X is diseased' will preserve truth if we impose a side-condition requiring that ' X ' refers to a rabid dog.²⁵ Presumably, however, we will not want to say that this means the argument is therefore 'restrictedly valid'.

²² Etchemendy 1999 (p. 28) calls these ranges 'grammatical categories', indicating his conception of them as syntactically defined classes. Barnes 2012a (p. 54) also notes that it is natural to assume that the specification of a logical form be syntactical only: 'I assume that questions of form are essentially questions of syntax – that formal features are either determined by or identical with syntactical features'.

²³ See Rini 2011 (pp. 26, 28, 54).

²⁴ I do not provide a precise definition of a semantic condition here. Roughly, I mean condition which cannot be determined to hold or not to hold of a term without knowing the meaning of that term. See Barnes 2012a for a wide-ranging treatment of this issue in a historical context.

²⁵ See MacFarlane 2000 (p. 38) for further discussion of the difficulties involved in using schemata to define a notion of validity.

Now it is not clear that Rini shows Aristotle's arguments to be restrictedly valid in anything but this trivially broad sense, since Rini does not show there is any C such that all of Aristotle's proofs are C -restrictedly valid. While the side-condition (7) validates the proof of Cesare LXL, different side-conditions are required for other syllogisms and their proofs. This is because the terms which are required to be 'red' differ, according to Rini, from syllogism to syllogism according to conversion rules which Aristotle employs in order to prove each syllogism.

For instance, when Rini comes to consider Aristotle's proof at *Prior Analytics* 31b12–19,²⁶ she finds that the A term needs to be 'red', with the consequence that this syllogism is not subject to the side-condition in (7) but rather the following:

$$\begin{aligned}\mathfrak{F} &= \{\forall, \neg, \supset, x, L, (,)\} \\ \mathfrak{V} &= \{A, B, C\} \\ \text{Ran}(B) &= \text{Ran}(C) = \{\text{Predicates in the language}\} \\ \text{Ran}(A) &= \{\text{Red predicates in the language}\}\end{aligned}\tag{8}$$

Then, when Rini 2011 (p. 101) discusses Disamis LXL, she finds Aristotle employing conversion rules which require C to be 'red', so that the argument is subject to yet a third side-condition, requiring the following modification:

$$\begin{aligned}\text{Ran}(B) &= \text{Ran}(A) = \{\text{Predicates in the language}\} \\ \text{Ran}(C) &= \{\text{Red predicates in the language}\}.\end{aligned}\tag{9}$$

Consequently, Rini does not provide a single sense of 'restrictedly valid' according to which all of Aristotle's modal proofs are 'restrictedly valid', except in the trivial sense discussed above. At best, Rini shows that for each of Aristotle's modal proofs, there is some sense of 'restrictedly valid' according to which it is valid. For these reasons, Rini 2011 does not provide us with persuasive reasons to call Aristotle's modal proofs valid. Instead, Rini effectively identifies the class of counterexamples to her formalizations of Aristotle's proofs and calls them valid on the grounds that they preserve truth so long as these counterexamples are left out of consideration.

This is unfortunate, since the project of finding a charitable interpretation of Aristotle's modal syllogistic based on a distinction between kinds of terms remains appealing given the persistent role that such distinctions played in traditional commentary.²⁷ For the remainder of this paper, I will consider one such interpretation, and argue that Robert Kilwardby's employment of term-kinds presents a way to interpret at least some of Aristotle's modal syllogisms and modal proofs as *formally* valid in the schematic sense.

3. Robert Kilwardby's interpretation of the *Prior Analytics*

Robert Kilwardby's circa 1240 commentary on the *Prior Analytics* consists of an exposition of each part of Aristotle's text, followed by an extended discussion in which he attempts to lay to rest the 'doubts [*dubia*]' of a student or interlocutor who raises questions concerning Aristotle's claims. Kilwardby works from the assumption that Aristotle is correct, and seeks an interpretation capable of vindicating both Aristotle's results and

²⁶ This syllogism is usually taken to be Disamis XLL, but Rini 2011 (pp. 97–100) questions this reading. The identity of the syllogism in question does not concern us here, rather only that the proof which Rini finds is not an instance of the schema with the side-condition given in (7).

²⁷ See Thom 2003 for a survey of traditional interpretations which rely on term-kind distinctions.

his arguments for them.²⁸ Like Rini, then, Kilwardby aims to show that Aristotle was a competent logician.

Kilwardby's attempt to show this is further similar to Rini's in its recourse to a distinction between two kinds of necessities. He distinguishes between 'per se' and 'per accidens' necessities, and takes conversion rules to apply only to the former.²⁹ This echoes Rini's distinction between 'genuine' and 'non-genuine' predications, only the latter of which Rini 2011 (p. 4) takes to be convertible. Furthermore, Kilwardby, again like Rini, employs a classification of terms in order to explain this difference. Kilwardby claims that convertible necessities must not have terms which are 'the name of an accident [*nomen accidentis*]' as their subject.³⁰ This, again, echoes Rini's claim that genuine predications must have 'red' terms as their subjects (Rini 2011, p. 4).

Rini compares her approach to Kilwardby's,³¹ and the two major interpretations of Kilwardby's logic to date (Thom 2007, Lagerlund 2000) support this comparison. Thom and Lagerlund, as I discuss below, both take Kilwardby to be placing restrictions on Aristotle's modal syllogisms in an ad hoc manner in order to secure the validity of mixed moods. I argue that this aspect of their interpretation is not well motivated, however, and that it is possible to give a more favourable reading of Kilwardby's treatment of the necessity syllogistic which preserves the formal validity of many of Aristotle's key results and proof methods.

I proceed as follows. First, I present Kilwardby's framework of term kinds and *per se* necessities, and his application of this framework to the problem of the Two Barbaras (3.1). I show how the readings of these passages due to Thom 2007 and Lagerlund 2000 burden Kilwardby's interpretation with problems parallel to those which I have raised in connection to Rini 2011 (3.2). I then reconstruct a semantics for necessities by analyzing Kilwardby's use of supposition theory in his solution to the problem of necessity conversion (3.3). In the appendix, I show that these semantics render Aristotle's conversion rules and the relevant first figure syllogisms formally valid in the schematic sense.

3.1. Kilwardby on necessity conversion

Kilwardby distinguishes between kinds of necessities in the course of discussing conversion rules for modal propositions. He considers putative counterexamples to Aristotle's rule that 'A is said necessarily of all B' converts to 'B is said necessarily of some A' (25a8–9). Someone might doubt this rule, Kilwardby observes, because it seems to yield the falsehood 'some humans are necessarily literate' from the true proposition 'everything literate is necessarily human'.³² Kilwardby mentions two responses to this doubt. I discuss his first response in Section 3.3. His second response, which is taken by many scholars to express his preferred solution,³³ involves distinguishing two grades of necessity, necessity *per se* and necessity *per accidens*:

Alternatively it can be said, quite plausibly, that propositions like this, which have the name of an accident as subject are not necessity-propositions *per se* but only *per accidens*. For a necessity-proposition *per se* requires the subject to be something of the predicate *per se* [*per se esse aliquid ipsius predicati*]. But when it is said

²⁸ Cf. Thom 2007 (pp. 3–5), Lagerlund 2000 (p. 42).

²⁹ Lectio 8 (dub.4). I discuss this passage in detail in Section 3.1. Here and throughout I cite the critical edition and translation by Paul Thom and John Scott (Kilwardby 2015). In places I adapt the translation so as to be more literal.

³⁰ Lectio 8:134 (dub.4).

³¹ See Rini 2011 (p. 5).

³² Lectio 8:13–18 (dub.4). He also considers there the apparent counterexample 'everything healthy, or awake, is necessarily an animal'. See Knuutila 2008 (pp. 538–544) for a discussion of these counterexamples and their history.

³³ Lagerlund 2000 (p. 28), 2008 (p. 309) and Knuutila 2008 (p. 539). But compare Thom 2007 (p. 89).

‘Everything literate of necessity is a human’, the subject is not something of the predicate *per se* [*non est aliquid per se ipsius predicati*]. But it is granted to be necessary because ‘literate’ is not separated from that which is something of ‘human’. But a necessity-proposition of this type is a *per accidens* necessity-proposition. So, when Aristotle teaches how to convert necessity-propositions, he only teaches how to convert necessity-propositions *per se*. The counter-examples that were put up are with *per accidens* necessity-propositions; and so all the counter-examples collapse.³⁴

This response grants the objector that not all necessity-propositions are convertible, but maintains that the conversion rules do hold for a certain kind of necessity-proposition which Kilwardby calls necessities ‘*per se*’, in contrast to inconvertible necessities ‘*per accidens*’. According to this response, although ‘all literate things are necessarily human’ is a necessity, it is not a necessity of the convertible type that Aristotle means to be discussing here. The response contains two characterizations of the difference between *per se* and *per accidens* necessities. First, a *per se* necessity-propositions ‘requires the subject to be something of the predicate *per se*’,³⁵ a condition Kilwardby takes the counterexamples to fail. This expression is explained in Kilwardby’s commentary on *Posterior Analytics*, to which he directs us for this usage of ‘*per se*’.³⁶ In his commentary on *Posterior Analytics* I.4, Kilwardby allows that propositions which are merely true at all times can be called necessary in so far as the terms are inseparable,³⁷ but he reserves a stronger sense of ‘necessity’ for those which exhibit one of the two relations that Aristotle calls ‘ $\chi\alpha\lambda\lambda\prime$ $\alpha\upsilon\tau\omicron$ ’ (73a34–73b2), rendered into Latin as ‘*per se*’.³⁸ Kilwardby interprets *per se* inherence as a definitional relation requiring one of the terms to occur in the definition of the other.³⁹ He associates the first sense of ‘*per se*’, which he seems to have primarily in mind here, with the relation of one term being placed under the other in the categorial order.⁴⁰ Since Porphyry, the relations of superior and inferior of terms in the categorial order had been represented as a genus-species tree.⁴¹ Propositions like ‘all humans are animals’ are *per se* in the first sense because ‘animal’ is part of genus-species the definition of ‘human’ or,

³⁴ Lectio 8:133 (dub.4). *Aliter etiam potest dici satis probabiliter, scilicet quod huiusmodi propositiones subicientes nomen accidentis non sunt per se de necessario sed per accidens tantum. Propositio enim per se de necessario exigit subiectum per se esse aliquid ipsius predicati. Cum autem dicitur ‘Omne grammaticum de necessitate est homo’, ipsum subiectum non est aliquid per se ipsius predicati. Sed quia grammaticum non separatur ab eo quod est aliquid ipsius hominis, ideo conceditur esse necessaria. Sed que sic est de necessario per accidens est de necessario. Quando ergo Aristoteles docet conuertere propositionem de necessario, solum docet conuertere propositiones que sunt de necessario per se. Instantia autem facta est in propositionibus que sunt per accidens de necessario; et sic pereunt omnes instantie.*

³⁵ Lectio 8:135–137 (dub.4).

³⁶ *Propositiones enim necessarie reducuntur ad aliquem modum inherendi per se, secundum quod dicit Aristoteles in primo Posteriorum, ‘Sola per se inherencia sunt necessaria’.* Lectio 9:458–461 (dub.9).

³⁷ Kilwardby notices the omnitemporality clause in Aristotle’s definition of $\chi\alpha\lambda\lambda\prime$ $\pi\alpha\nu\tau\omicron\varsigma$ predications (*Posterior Analytics* 73a28–29, Ross 1949 edition used here and throughout) and concludes that $\chi\alpha\lambda\lambda\prime$ $\pi\alpha\nu\tau\omicron\varsigma$ denotes a strictly weaker, non-definitional form of necessity, and identifies these with inseparable accidents. See Cannone 2002 (pp. 118:66–69, 119:83–90); see Knuutila 2008 (pp. 529–530) and van Rijen 1989 (pp. 133–137) for the history of associating necessities which are not *per se* with inseparable accidents. Cf. Nortmann 1996 (p. 36) for a modern interpretation which also connects Aristotle’s omnitemporality clause with the truth conditions for necessities.

³⁸ Cannone 2002 (p. 119:83–89).

³⁹ Cannone 2002 (p. 120:107–108, 121:139–140).

⁴⁰ On this, see Thom 2007 (pp. 69–71, 157) and Kilwardby 2015 (p. xxiii). Later in his commentary, Kilwardby also makes use of the second sense of *per se*, but appears to limit its sphere of applicability to the contingency syllogistic (Lectio 20:707).

⁴¹ See, for example, Peter of Spain 2014 (p. 137). Copenhagen, Normore and Parsons report there (footnote 8) that Porphyry himself mentions no diagram, but describes higher genera as ‘branching [*ramosus*]’ into lower ones, while Boethius’s translation mentions a ‘figure that provides a visual example [*descriptio sub oculis ponat exemplum*]’. Lagerlund 2000 (p. 32) also takes Kilwardby’s syllogistic necessities to express ‘essential properties of things located in a genus-species structure’.

equivalently, because ‘human’ falls under ‘animal’ in the categorial order. On the other hand, ‘everything literate is necessarily human’ is a necessity, but not in the same sense. ‘Literate’ does not fall under ‘human’ as a species on the genus-species tree. This proposition is rather deemed to be a necessity ‘because the literate is not separated from that which is something of the human’.⁴²

Second, this solution invokes a distinction between terms which are ‘the name of an accident [*nomen accidentis*]⁴³ and those which are not. Only necessities whose subject term is not ‘the name of an accident’ are *per se* necessities. This distinction between kinds of terms is also elaborated in Kilwardby’s commentary on *Posterior Analytics*. Kilwardby holds that the expression ‘*per se*’ can be used to qualify not only propositions but also terms. Whereas Aristotle’s first, second and fourth senses of ‘*per se*’ are relational modes of ‘inherence [*inherendi*]⁴⁴ and hence qualify ‘one entity in relation to another’,⁴⁵ Aristotle’s third mode of *per se* qualifies an entity ‘absolutely [*absolute*]⁴⁶. It is a mode of *per se* ‘being [*essendi*]⁴⁷ rather than *per se* inherence,⁴⁷ and hence it can be taken to qualify *terms* rather than *predications*.

A term is *per se* in this sense if it signifies a substance.⁴⁸ Predications whose subjects are substance terms are what Kilwardby later in his commentary calls predications ‘*secundum se*’, as opposed to predications ‘*secundum accidens*’.⁴⁹ By excluding *per accidens* necessities on the grounds that their subject is ‘the name of an accident’, Kilwardby is distinguishing *per se* necessities as those necessities which are *secundum se*.

Kilwardby’s response to the objection, then, is to admit that not all necessities are convertible, but to claim that Aristotle is talking about a special kind of necessity – necessity *per se* – for which the conversion rules do hold. How are we to interpret this distinction? One option is to take Kilwardby to be disambiguating Aristotle’s expression ‘of necessity’.⁵⁰ He would then be distinguishing two sorts of propositions, necessities and necessities *per se*, at the syntactic level. Kilwardby could then be read as giving two characterizations of the truth conditions for propositions containing the words ‘of necessity’ in the stronger, *per se* sense. First, a *per se* necessity is true if and only if it does not have the name of an accident as its subject, and the corresponding plain necessity is true. Writing a_L for ‘is said of all of necessity’, a_L for ‘is said of all of necessity *per se*’,⁵¹ and using σB to mean that B is a *per se* term, we can write:

$$Aa_L B \text{ is true if, and only if } \sigma B \text{ and } Aa_L B. \quad (10)$$

To say that in such propositions the subject is ‘something of the predicate *per se* [*aliquid per se ipsius predicati*]⁵² would then be to give a second characterization of this truth

⁴² *non separatur ab eo quod est aliquid ipsius hominis*. Lectio 8:139–140 (dub.4).

⁴³ Lectio 8:134 (dub.4).

⁴⁴ *Cannone 2002* (p. 127:292–293).

⁴⁵ *entis ordinati ad alterum*. *Cannone 2002* (p. 130:375).

⁴⁶ *Cannone 2002* (p. 120:120).

⁴⁷ *Cannone 2002* (p. 122:169–171).

⁴⁸ [...] *hoc autem modo est substantia per se et maxime prima substantia* (*Cannone 2002*, p. 122:171–173).

⁴⁹ See *Cannone 2003–2004* (p. 212:74–76). Like Philoponus, Kilwardby takes Aristotle’s theory of demonstration not to be concerned with ‘unnatural’ predications which predicate a subject of an accident (compare *Cannone 2003–2004*, p. 228:78–80 and *Philoponus 2012*, p. 237:13–25). Instead, Kilwardby takes demonstrations to be concerned with predications of ‘a superior of an inferior’ (as in a first mode *per se* predication) or an ‘accident of a subject’ (as in a *per se* predication in the second mode) (*Cannone 2003–2004*, p. 228:92), both of which require the subject to be a *per se* term. Here Kilwardby appears to be applying the same requirement to the modal syllogistic in the *Prior Analytics*.

⁵⁰ That is, ἐξ ἀνάγκης (25a32), or *ex necessitate* in Latin.

⁵¹ For a defense of the interpretation of Aristotle’s modalities as modifiers of the copula, see *Patterson 1995* (pp. 15–23). In Kilwardby’s time, there was a precedent for this approach in the anonymous *Dialectica Monacensis*. See *De Rijk 1967* (p. 478:15).

condition:

$$Aa_L B \text{ is true if, and only if } A \text{ stands to } B \text{ in a relation of} \\ \textit{per se} \text{ inference (of the first kind).} \quad (11)$$

Kilwardby's interpretation would then be that Aristotle's claim that necessities convert is true when 'necessity' is understood in the stronger, definitional sense captured by (10) and (11), but false when it is understood in the weaker sense of inseparability.

On the other hand, we could take Kilwardby to recognize only a single sense of 'of necessity', but to be claiming that necessities are only convertible given semantic conditions not required for their truth: Namely, that the subject is not 'the name of an accident' and stands to the predicate in a relation of *per se* inference. To call a proposition a necessity '*per se*' would then not be to characterize its meaning, but rather its truthmaker under a given interpretation:⁵² A necessity is *per se* when its subject is not the name of an accident and this subject is *aliquid per se ipsius predicati*.

On the first reading, Kilwardby is distinguishing two syntactic forms of proposition, necessities and necessities *per se*. Kilwardby's remarks could then be read as outlining a semantics for these propositions, and one could ask whether this semantics validates necessity conversion in a manner consonant with their syllogistic behaviour. On the other hand, if Kilwardby takes being *per se* to be a status accorded to a generic necessity when it is made true by a *per se* predication, then his requirement that necessities in first figure syllogisms be *per se* would be a semantic restriction, and consequently could not be used to define a non-restrictive side-condition or a schematic notion of formal validity as defined on p. 7.

3.2. The readings of Thom and Lagerlund

Thom 2007 and Lagerlund 2000 both offer interpretations of the second type. As Thom 2007 (p. 21) reads him, Kilwardby takes it to be characteristic of *per se* terms that they apply necessarily to whatever they apply to, in both a *de dicto* and a *de re* sense, a condition which we can represent in modal predicate logic as follows:

$$L\forall x(\phi x \equiv L\phi x). \quad (12)$$

As this representation makes clear, Thom's condition for a term's being *per se* is equivalent to Rini's 'Substance Principle' with an added wide-scope necessity operator (see p. 2 of this paper).⁵³ Thom 2007 (p. 21) reads Kilwardby as taking syllogistic necessities to express that a *de dicto* necessity holds between *per se* terms. That is, he takes the syllogistic necessity $Aa_L B$ to express that:

$$L\forall x(Bx \supset Ax) \wedge L\forall x(Ax \equiv LAx) \wedge L\forall x(Bx \equiv LBx) \quad (13)$$

while the syllogistic necessity $Ai_L B$ expresses that:

$$L\exists x(Bx \wedge Ax) \wedge L\forall x(Ax \equiv LAx) \wedge L\forall x(Bx \equiv LBx). \quad (14)$$

These semantics straightforwardly suffice to explain why Barbara LLL and Darii LLL are valid, and it is easy to see that the a_L - and i_L -conversion rules hold given definitions (13)

⁵² Compare Thom 2007 (p. 158).

⁵³ Thom does not explicitly state the right-to-left implication, but I am assuming he takes it to be trivial that what is necessarily ϕ is ϕ . Hence, I write \equiv rather than \supset in (12), although the latter more closely follows Thom's formulation. Thom 2007 also does not make clear whether he takes the *de dicto* modality to fall inside or outside the scope of the quantifier. The difference turns out not to be important, since Thom does not take Kilwardby to be making use of the logic of this condition at all (see this section below). See Nortmann 1996 for a contemporary reading which seeks to validate Aristotle's modal syllogisms using similar constructions.

and (14). However, this interpretation saddles Kilwardby with the converse of the problem with Rini's analysis of necessities as *de re* modals. Whereas Rini's definition validates mixed-mood syllogisms but not conversion, Kilwardby's, on Thom's reading, validates conversion but not Barbara LXL, since the argument:

$$\frac{L\forall x(Bx \supset Ax) \wedge L\forall x(Ax \equiv LAx) \wedge L\forall x(Bx \equiv LBx) \quad \forall x(Cx \supset Bx)}{L\forall x(Cx \supset Ax) \wedge L\forall x(Ax \equiv LAx) \wedge L\forall x(Cx \equiv LCx)} \quad (15)$$

is not formally valid under any obvious logic for *L*.⁵⁴

Kilwardby does attempt to explain why Barbara LXL is valid but Barbara XLL is not. However, as construed by the two major interpretations to date (*Thom 2007* and *Lagerlund 2000*), his attempt is not very successful. Kilwardby claims that a necessity major in a first figure syllogism 'appropriates' the minor to be a '*simpliciter*' assertoric, but a necessity minor does not 'appropriate' an assertoric major:

[...] when the major is a necessity it appropriates the minor to itself in such a way that the latter has to be a *simpliciter* assertoric and the minor extreme has to be taken essentially under the middle, in such a way that the minor is in reality necessary. [...] But when the major premise is assertoric (and the minor cannot appropriate the major to itself but the other way round), the major does not have to be a *simpliciter* assertoric but may well be an as-of-now assertoric.⁵⁵

Two pieces of terminology call for explanation: the adjective '*simpliciter*' which Kilwardby uses to qualify 'assertoric' and the verb 'appropriate [*appropriare*]'. As Lagerlund reads him, Kilwardby defines being a *simpliciter* premise of a mixed-mood necessity syllogism in such a way that there is 'no difference between a necessity proposition and a *de inesse simpliciter* proposition' (*Lagerlund 2000*, p. 40). To say that the major 'appropriates' the minor to be necessary, on Lagerlund's reading, is just to say that the minor of a first figure mixed necessity syllogism is to be read as a necessity. This gives a straightforward, if unedifying explanation of the validity of Barbara LXL, under the assumption that Barbara LLL is valid: The assertoric minor is to be read as a necessity, and hence Barbara LXL is simply Barbara LLL by a different name.⁵⁶

Unlike Lagerlund, *Thom 2007* (pp. 37–38) does not take Kilwardby to be claiming that Barbara LXL represents the same mood as Barbara LLL. Instead, Thom claims, Kilwardby takes instances of syllogisms to be ill-formed which have a necessity major and an assertoric minor that is not necessarily true. The minor of any well-formed first figure LXL syllogism must rather be a necessarily true assertoric (*Thom 2007*, p. 38). This is what it means, on Thom's reading, to say that the major 'appropriates' the minor to be an 'unrestricted' (or *simpliciter*) assertoric (*Thom 2007*, p. 161). In this way, the truth of the conclusion of any well-formed Barbara LXL syllogism is secured by the validity of Barbara LLL (since the formation rules guarantee that the minor is necessarily true), even though the minor premise of Barbara LXL does not need to be explicitly modalized. On the other hand, Kilwardby takes no such formation restrictions to apply to syllogisms which

⁵⁴ Such a logic for *L* would presumably need to make the premises entail that *C* is a *per se* term, that is, one for which $L\forall x(Cx \equiv LCx)$. Thom does not explore whether or how his interpretation might give rise to such a logic.

⁵⁵ [...] *cum maior sit de necessario appropriat sibi minorem ita quod oportet ipsam esse de inesse simpliciter et minorem extremitatem accipi essentialiter sub medio, ita quod minor sit necessaria secundum rem. [...] Maior autem cum sit de inesse (et minor non potest appropriare sibi maiorem, sed econverso), non oportet illam maiorem esse de inesse simpliciter sed bene poterit esse de inesse ut nunc.* Lectio 15:257–265 (dub.7). Cf. *Thom 2007* (p. 209).

⁵⁶ Cf. *Lagerlund 2000* (p. 41).

have an assertoric major and a necessity minor (Thom 2007, p. 161). Hence, Barbara LLL does not secure the validity of Barbara XLL, because the major is not required by the formation rules to be a *simpliciter* assertoric.

Thom 2013 concludes that Kilwardby was not attempting to establish that Aristotle's syllogisms are formally valid in the modern sense.⁵⁷ Hence, although Kilwardby explicitly tells us that he takes Aristotle to be 'talking about the syllogism according to its form insofar as it abstracts from matter' in the *Prior Analytics*,⁵⁸ Thom 2013 warns against identifying Kilwardby's notion of logical 'form' with the modern one.⁵⁹

As well as making Kilwardby's solution disappointing, there are textual reasons to question this conclusion. Thom, as we have seen, takes *per se* necessities to express that a *de dicto* necessity holds between *per se* terms. However, as we have also seen, in his *Posterior Analytics* commentary Kilwardby takes *per se* propositions to entail a relationship between the *definitions* of terms. The difference is significant, since Kilwardby explicitly states that not all necessities are definitional. Hence, Thom's interpretation of the truth condition for *per se* necessities appears to be too weak. Furthermore, Kilwardby's discussion of necessity in his commentary on the *Posterior Analytics* clarifies that he takes the phrase 'of necessity' to be 'said in two ways [*dicitur dupliciter*]', thus favouring a reading of the first kind outlined on p. 11 of this paper – that is, a reading on which Kilwardby is disambiguating between two senses of 'of necessity'.⁶⁰ Let us therefore turn to Kilwardby's discussion of necessities under the assumption that he is outlining the semantics for an alternative sense of 'necessity', and consider how this fares as an attempt to achieve agreement with Aristotle's results.

3.3. *Supposition theory and the semantics of necessities*

The alternative reading takes Kilwardby's discussion of *per se* necessities to provide two characterizations of a single truth condition for *per se* necessities, viz. (10) and (11). In order for this reading to be plausible, we need to explain why Kilwardby might take these two truth conditions to be equivalent, and how they can be understood to specify a semantics for necessities intended to obey convertibility and give the desired results with regard to the Two Barbaras. For this purpose, we need first to clarify what Kilwardby takes to be the truth condition for non-convertible necessities that express mere inseparability.

Immediately before claiming that conversion rules apply only to *per se* necessities, Kilwardby gives the following explanation for why the proposition 'everything literate is necessarily human' fails to convert:

⁵⁷ Thom 2007 (p. 5) warns that there are 'important elements in Kilwardby's text' which 'militate against' reading Kilwardby as putting forward 'a formal system of syllogistic in the modern sense'. Nevertheless, Thom 2007 (p. 243) claims that 'Kilwardby's appropriation system can be given a formal characterization, albeit an *ad hoc* one'.

⁵⁸ *Intendit enim Aristoteles in hoc libro de syllogismo secundum formam prout abstrahit a materia*. Lectio 14:107–8 (dub. 1).

⁵⁹ Instead, Thom claims, Kilwardby only means to systematize Aristotle's presentation by enumerating a set of 'principles' which summarize the syllogisms Aristotle takes to be valid. For instance, the following 'principles' apply to mixed mood necessity syllogisms (Thom 2013, p. 152):

(P8) in first figure assertoric/necessity syllogisms, the necessity-proposition must be major.

(P9) in second figure assertoric/necessity syllogisms, one premise must be a universal negative necessity-proposition.

(P10) in affirmative third figure assertoric/necessity syllogisms, the necessity-premise must be a universal affirmative.

(P11) in negative third figure assertoric/necessity syllogisms, the necessity-premise must be a universal negative.

On Thom's reading, Kilwardby claims that a first figure syllogism has 'appropriate syllogistic form' exactly when it conforms to all of these principles, even though there are perfectible syllogisms which do not fall under the scope of these principles (Thom 2013, pp. 159–160).

⁶⁰ *Item, nota quod 'ex necessitate inesse' dicitur dupliciter*. (Cannone 2002, p. 119:83).

To this it can be said that the conversion is blocked because of the different ways of taking the terms in subject-position and in predicate-position. For when it is said ‘Everything literate of necessity is a human’, the term ‘literate’ stands for its *suppositum*; for if it were to stand for the quality cosignified by the name the proposition would be false. However, when it is stated conversely ‘Some human of necessity is literate’, the term ‘literate’ is taken for the quality alone. *For such is the nature of predicate- and subject-terms, that when they are subjects they stand for supposita, but when they are predicated they stand for a quality and a form, and these different ways of taking terms like ‘literate’ in subject- and predicate-position obstructs the conversion.*⁶¹

Kilwardby is appealing to supposition theory, the medieval theory which classified the ways a term could be interpreted depending on its propositional, inferential and dialogical context.⁶² He proceeds by explaining what would be required to make the converted and the unconverted necessity propositions here respectively true. Hence, we can read him as using the theory of supposition to provide the truth conditions for necessities in a broad sense, including those such as ‘all literate things are necessarily human’, which are not required to express a relation of *per se* inherence.⁶³

In order to understand Kilwardby’s solution, it will help to go beyond Kilwardby’s commentary and draw on some contemporaneous presentations of *suppositio*. Other authors writing at the time, such as Lambert of Auxerre, associate every term with two distinct entities. First, each term is associated with a ‘thing the term is imposed to signify’.⁶⁴ Secondly, every term is associated with the ‘supposita [logically] contained under that [signified] thing’.⁶⁵ The *supposita* associated with a term are the individuals or the collection of individuals that have the quality signified by that term.⁶⁶ The ‘thing the term is imposed to signify’, also called the term’s *signification*, however, does not refer to any individual or collection of individuals. Lambert explains using the example of the term ‘*homo*’:

For example, the signification [*significatio*]⁶⁷ of ‘man’ extends only to *man*, not to the things contained under *man*; for man signifies *man*, not Socrates or Plato (Lambert of Auxerre 2015, p. 255).

⁶¹ *Ad hoc potest dici quod impeditur conuersio per diuersam acceptionem terminorum ex parte subiecti et ex parte predi-
cati. Cum enim dicitur ‘Omne grammaticum de necessitate est homo’, iste terminus ‘grammaticum’ stat pro supposito; si
enim staret pro qualitate consignificata per nomen, esset propositio falsa. Cum autem dicitur econuerso ‘Aliquis homo de
necessitate est grammaticus’, accipitur iste terminus ‘grammaticum’ pro qualitate sola. Talis enim est natura terminorum
predicatorum et subiectorum, quod cum subiciuntur stant pro suppositis, cum predicantur stant pro qualitate et forma. Et
hec diuersa acceptio huius termini ‘grammaticum’ ex parte subiecti et predi-
cati prohibet conuersionem.* Lectio 8:119–131 (dub.4). My emphasis.

⁶² See Ebbesen 1981, 2013 and Parsons 2008. See also Dutilh-Novaes 2007 for a sophisticated treatment of later versions of the theory.

⁶³ Dutilh-Novaes 2007 (p. 44) denies that *suppositio* theory concerns truth conditions, as I take it to in what follows. Her interpretation, however, is informed mainly by fourteenth-century theories, centrally Ockham’s, and she admits that earlier theorists such as William of Sherwood may have understood *suppositio* differently (p. 44). Copenhaver, Normore and Parsons claim that Peter of Spain’s theory of supposition develops ‘something like a theory of truth conditions’ (Peter of Spain 2014, p. 81), while Ebbesen 1981 (p. 41) contends that with early supposition theory ‘the medievals got a method of stating the truth-conditions of sentences’.

⁶⁴ See Lambert of Auxerre 2015 (p. 255). Here I use the translation of Lambert of Auxerre 1988 (p. 105), which is more perspicuous than Maloney’s ‘the [signified] thing for which a term is imposed’. Lambert’s expression is ‘re[s] ad quam significandam imponitur terminus’ (Lambert of Auxerre 1971, p. 206). Often, Lambert abbreviates this to ‘res significata’ or simply ‘res’.

⁶⁵ Lambert of Auxerre 2015 (p. 255). Lambert’s expression here is ‘supposita contenta sub illa re’ (Lambert of Auxerre 1971, p. 206, following Maloney’s reading of *contenta* against *contempta*).

⁶⁶ Lambert of Auxerre 2015 (p. 255). Lambert refers to *De Interpretatione* 16a3–5 (Lambert of Auxerre 2015, p. 253) and *De Anima* III.8, 431b30–432a1 (Lambert of Auxerre 2015, p. 254) in support of his semantic theory.

⁶⁷ Lambert of Auxerre 1971 (p. 206).

What is the *man* which ‘man’ signifies, but which is neither Socrates nor Plato nor anyone else? Lambert conceives of the signification of a term as an objective entity, a thing ‘existing outside the soul’ (*Lambert of Auxerre 2015*, p. 254). As opposed to later approaches, which tended to construe signification as a concept or an item of mental language,⁶⁸ Lambert seems to think of the signification of a word as an objectively existing intensional entity – in this case, the form associated with the word ‘man’ (*Lambert of Auxerre 2015*, p. 259).⁶⁹

Lambert holds that a term can stand for either its signification or its *supposita* depending on the context in which it occurs. When a term is interpreted to stand for its *suppositum* or *supposita*, it is said to have ‘personal supposition [*suppositio personalis*]’; when it stands for its signification it is said to have ‘simple supposition [*suppositio simplex*]’.⁷⁰ Here, Kilwardby is proposing the following rule of supposition for necessities: Terms in subject position have personal supposition, while terms in predicate position have simple supposition.⁷¹ That is, a term in subject position must be interpreted for the class of individuals associated with it, whereas a term in predicate position must be interpreted for the form that is associated with it.⁷² Hence the asymmetry in meaning: In the proposition ‘Everything literate of necessity is a human’, the term ‘literate’ is in subject position, and consequently serves to pick out a collection of individuals (the literate individuals), which Kilwardby calls the *supposita*. On the other hand, in the proposition ‘Some human of necessity is literate’, the term ‘literate’ serves to pick out a form, the form of literacy. Kilwardby claims that the connection expressed in a necessity proposition between the form of humanity and all literate individuals holds, and hence the unconverted propositions is true, whereas the same connection does not hold between the form of being literate and some human individuals, and hence the converted proposition is false.

The supposition theory solution makes recourse not only to the individuals denoted by a term, but also to the form associated with a term. In order to reflect this, an adequate formalization of the structure relative to which Kilwardby’s necessity propositions are evaluated should include not only a domain of individuals, but also a domain of forms. Let us write these as *I* and *F* respectively. We can then formalize the two supposition-theoretic notions, the signification and *supposita* associated with each term, as two functions mapping from

⁶⁸ See Parsons 2008 (pp. 186–187).

⁶⁹ See also Lambert of Auxerre 1988 (p. 107): ‘[...] when one says “White distinguishes [*album disgregat*]”, “white” is interpreted here for itself [...] or for its [signified] thing [*pro re*] [...]; for that predicate applies to white not by reason of a *suppositum* but by reason of its form [*ratione sue forme*]’ (*Lambert of Auxerre 2015*, p. 256, appears to take this example differently). Cf. Parsons 2008 (p. 198).

⁷⁰ *Lambert of Auxerre 2015* (pp. 258–259). Confusingly, ‘*suppositio*’ is used by Lambert both for (i) the way that a term is interpreted in context, and (ii) the class of individuals associated with a term. Hence, on Lambert’s usage, a term does not always supposit for its *supposita* (*Lambert of Auxerre 2015*, p. 255). To avoid reproducing this confusion here, I will use ‘supposition’ to refer to the way a term refers in context, and reserve ‘*supposita*’ and ‘*suppositum*’ for the individuals or individual associated with a term.

⁷¹ Here I am taking Kilwardby to be suggesting this as a rule of supposition for necessity-propositions rather than all propositions. The text does not make clear what scope Kilwardby intends this rule to have. My main reason for choosing to take this as a claim about necessity propositions only is charity: Coupling this with an extensional interpretation of assertorics allows, as I go on to discuss, a semantics for necessities to be formulated which captures a number of Aristotle’s results in the necessity syllogistic. If my interpretation is right, then Kilwardby’s use of supposition theory here may provide a counter-example to Ebbesen’s claim that supposition theory had become ‘a sort of dead knowledge’ by the second half of the thirteenth century and that ‘nowhere in the Parisian works from the latter half of the century is the theory and its rules really used for any serious purpose’. See Ebbesen 1981 (p. 44).

⁷² A theory of multiple denotation might better capture the idea that a term can stand for multiple *supposita*. On this, see Dutilh-Novaes 2007 (pp. 55–56). For our purposes, it will suffice to treat the *supposita* of a term as a set of individuals.

terms to subsets of individuals and forms, respectively:

$$\begin{aligned} \text{Sup} : \mathcal{T} &\rightarrow \mathcal{F} \\ \text{Con} : \mathcal{T} &\rightarrow \mathcal{P}(I). \end{aligned} \tag{16}$$

In addition to the set of individuals and forms associated with terms, Kilwardby employs the notion of an *individual* bearing a given form.⁷³ Kilwardby takes individuals to possess multiple forms, ordered into a hierarchy.⁷⁴ This suggests formalizing the forms associated with each individual as a set or a sequence. However, Kilwardby takes each individual to have a unique ‘completive [*completiva*]’ form, which is subordinated in the categorial order to all and only those other forms that the individual bears.⁷⁵ Let us therefore introduce, alongside *Con* and *Sup*, a mapping from individuals to forms, and call this *Form*:

$$\text{Form} : \mathcal{I} \rightarrow \mathcal{F},$$

where $\text{Form}(x) = C$ means that C is the completive form of x . Since an individual’s completive form is subordinated to all and only those forms that individual bears, we can express ‘ x has form f ’ as

$$\text{Form}(x) \leq f,$$

where \leq is the relation of one form being subordinated to another in the categorial order.

Let us now consider how to represent Kilwardby’s truth conditions for affirmative necessities using this framework. Kilwardby claims that Aa_LB is true just if every individual supposed by the subject term (i.e. every member x of $\text{Sup}(B)$) bears the form that is cosignified by the predicate (that is, $\text{Form}(x)$ falls under $\text{Con}(A)$). Correspondingly, a universal particular necessity says that some individual supposed by the subject term (some member x of $\text{Sup}(B)$) has a form falling under the cosignification of the predicate. We take affirmative universal necessities to have existential import.⁷⁶ This then gives:

$$\begin{aligned} Aa_LB &\text{ is true if, and only if } \text{Sup}(B) \neq \emptyset \text{ and } \forall x \in \text{Sup}(B) : \text{Form}(x) \leq \text{Con}(A) \\ Ai_LB &\text{ is true if, and only if } \exists x \in \text{Sup}(B) : \text{Form}(x) \leq \text{Con}(A). \end{aligned} \tag{17}$$

Kilwardby’s claims regarding negative necessities are complex, and it is not clear that he takes them to be analyzable using the same framework.⁷⁷ However, a natural way to extend the same ideas to negative propositions would be as follows:

$$\begin{aligned} Ae_LB &\text{ is true if, and only if } \neg \exists x \in \text{Sup}(B) : \text{Form}(x) \leq \text{Con}(A), \\ Ao_LB &\text{ is true if, and only if } \neg \forall x \in \text{Sup}(B) : \text{Form}(x) \leq \text{Con}(A). \end{aligned} \tag{18}$$

These can then be viewed as the truth conditions for necessities in the broad sense that includes necessities made true merely by inseparability. A necessity in the strict, *per se* sense differs in that it furthermore requires the subject term not to be ‘the name of an accident’. Terms which are ‘the name of an accident’ pick out their *supposita* by naming

⁷³ See Ebbesen 2013 (pp. 60–61), who argues that the idea of an individual bearing a form was a key concept in early supposition theory.

⁷⁴ He argues, for instance, that the human soul is constituted of ‘three substantial forms (*tribus formis substancialibus*)’. See Silva 2012 (pp. 77, 88).

⁷⁵ The intellectual form of the soul, for instance, occupies this privileged position among the multiplicity of forms which the soul possesses. It ‘completes’ all of the other forms but they do not ‘complete’ it. See Silva 2012, pp. 85–86.

⁷⁶ The same assumption is *not* made for necessities *per se*, since Kilwardby appears not to take these to imply the existence of any individuals named by the subject term (see below).

⁷⁷ See Thom 2007 (pp. 26–28).

an accident they share rather than one of their forms.⁷⁸ That is, the *supposita* of a *per accidens* term does *not* have the form which is signified by that term. Conversely, a term which is not the name of an accident, or *per se*, is a term like ‘homo’, which indicates the ‘nature or substance’ of its *supposita*.⁷⁹ In the language of supposition theory, this is to say that *supposita* of *per se* terms have the form of that term’s cosignification.⁸⁰ Using the notation we have been developing, we can therefore define a term’s being ‘not the name of an accident’ or *per se* as follows:

$$\sigma T \text{ is true if, and only if, } \forall x \in \text{Sup}(T): \text{Form}(x) \leq \text{Con}(T). \quad (19)$$

Hence, the truth-conditions for *per se* affirmative necessities, defined as necessities whose subject term is *per se*, are as follows:

$$Aa_{\perp}B \text{ is true if, and only if } \forall x \in \text{Sup}(B): \text{Form}(x) \leq \text{Con}(A), \text{ and} \quad (20)$$

$$\forall x \in \text{Sup}(B): \text{Form}(x) \leq \text{Con}(B),$$

$$Ai_{\perp}B \text{ is true if, and only if } \exists x \in \text{Sup}(B): \text{Form}(x) \leq \text{Con}(A), \text{ and} \quad (21)$$

$$\forall x \in \text{Sup}(B): \text{Form}(x) \leq \text{Con}(B).$$

We have seen that Kilwardby also characterizes *per se* necessities as those which express a relation of *per se* inherence in Aristotle’s first sense. *Per se* inherence, we said above, is a definitional relation that holds when the definition of the predicate term occurs in the definition of the subject term. The definitions in question are those associated with the terms themselves, rather than the individuals which fall under them, and hence the relation of *per se* inherence does not depend on which individuals named by the term are in existence.⁸¹ The fact that *per se* necessity does not depend on any individuals referred to by the term means that *per se* inherence is a relation which holds between the cosignifications, rather than the *supposita*, of subject and predicate, since the cosignification of a term represents a form timelessly associated with it, independently of which individuals described by it actually exist. Accordingly, we should formalize *per se* inherence as a direct relation of categorial subordination between the cosignification of two terms. I will follow Thom 2007 (p. 21) in assuming that *per se* inherence furthermore requires both terms to be *per se*. We can then represent Kilwardby’s claim that subject and predicate stand in a relation of *per se* inherence, in the relevant sense, as:

$$\sigma A, \sigma B \text{ and } \text{Con}(B) \leq \text{Con}(A) \quad (22)$$

and so, substituting this back into (11):

$$Aa_{\perp}B \text{ is true if, and only if } \sigma A, \sigma B \text{ and } \text{Con}(B) \leq \text{Con}(A) \quad (23)$$

⁷⁸ Cf. Ebbesen 1981 (pp. 35, 41).

⁷⁹ Cannone 2002 (p. 123:185–188). See also Cannone 2003–2004 (pp. 228:70–71, 229:116–117).

⁸⁰ A similar construction is found in William of Sherwood, who uses it to describe personal supposition. William explains that personal supposition is produced when ‘a thing bearing the form signified by the name is supposed [*supponitur res deferens formam significatam per nomen*]’ (William of Sherwood 1966, p. 110).

⁸¹ Kilwardby makes it clear that he takes animal to inhere *per se* in human, for instance, regardless of which individual humans exist at any given time. See Lectio 15 (dub.7). Kilwardby’s view that necessities do not depend on the actual existence of individuals to which they refer is also reflected in his 1277 condemnations. One proposition Kilwardby condemns is ‘That necessary truth [always] depends on the persistence of the subject [*Item quod veritas cum necessitate tantum est cum constancia subjecti*.]’ (Uckelman 2010, p. 217). Any universal affirmative *per se* necessity is an example, for Kilwardby, of a necessity whose truth does not depend on the persistence of individual subjects.

(23) and (20) give two apparently different definitions of *per se* necessity. Under some reasonable assumptions, however, it can be shown that (20) and (23) are equivalent for non-empty terms.⁸² Similarly, it can be shown that under the same conditions (21) is equivalent to the condition that:

$$\sigma A, \sigma B \text{ and } (Con(B) \leq Con(A) \text{ or } Con(A) \leq Con(B)).^{83} \quad (24)$$

Similarly, it can be shown that requiring terms to be non-empty and the subject term to be *per se* renders particular and universal negative necessities, as defined above, equivalent to the following conditions:

$$\sigma A, \sigma B \text{ and } Con(B) \not\leq Con(A), \quad (25)$$

$$\sigma A, \sigma B \text{ and } (Con(B) \not\leq Con(A) \text{ and } Con(A) \not\leq Con(B)). \quad (26)$$

Two important results follow from this. First, contrary to what some scholars have claimed, the semantics Kilwardby assumes for necessities of the convertible and non-convertible types in his two solutions are equivalent. The solution in terms of *per se* necessities distinguishes a stronger and weaker form of necessity using native Aristotelian concepts. The solution in terms of supposition theory explicates these conditions using a more modern theory which, while not itself part of the Aristotelian conceptual repertoire, is well suited to explicate Aristotle's concepts of *per se* terms and *per se* inference. Second, these results show that Kilwardby's claim that necessities with a *per se* subject term convert is correct. The representation of the truth condition for a *secundum se* necessity in terms of *per se* inference shows that it can only be made true by a relation of categorial subordination holding between the signification of its terms. Hence, a *per se* particular affirmative necessity turns out to be equivalent to a disjunction of two universal necessities, from which the desired conversion result readily follows.⁸⁴ The representation of Kilwardby's truth conditions in terms of supposition theory also allows us to see how these provide a solution to the problem of the Two Barbaras.

3.4. Appropriation again

So far, we have found that Kilwardby's supposition semantics allow us to formulate a definition of necessity which obeys conversion. Does it also display the required syllogistic behaviour?

Let us return to Kilwardby's doctrine of 'appropriation':

[...] when the major is a necessity it appropriates the minor to itself in such a way that it has to be a *simpliciter* assertoric and the minor extreme has to be taken essentially under the middle, in such a way that the minor is in reality necessary. But

⁸² The proof is given in the [appendix](#). See theorem A.12. The proof relies on the following observation. If each *suppositum* of the subject term also bears the form signified by the subject (i.e. if the subject term is *per se*), then the forms of these *supposita* fall under anything which the cosignification of the subject term itself does, and vice versa. If the set of the subject's *supposita* is non-empty, therefore, we can bypass consideration of the individuals supposed by the subject, and simply look to see whether the signification of the subject term falls under the signification of the predicate term.

⁸³ This is proven in Section A.12. This representation shows that Kilwardby's definition of *per se* terms and necessities causes a disjunctive analysis to apply to particular necessities, as in *Thom 1996* (p. 146) and *Malink 2013* (p. 179). However, unlike Thom and Malink, Kilwardby does not stipulate a disjunctive reading of particular necessities. Instead, this disjunctive representation is a consequence of his *suppositio*-theoretic definitions of *per se* terms and necessity. I take this to be a significant advantage of Kilwardby's approach. Kilwardby seems to have been aware that at least something like this was a consequence of his definitions; see *Lectio 9* (dub.9).

⁸⁴ See [appendix](#), Section A.5.

when the major is assertoric (and the minor cannot appropriate the major to itself but the other way round), the major does not have to be a *simpliciter* assertoric but could well be an as-of-now assertoric.⁸⁵

Kilwardby's claim follows if we take him to be assuming that all predications (assertoric and necessary) in the necessity syllogistic are *secundum se*.⁸⁶ This requirement forms a part of the truth conditions for the propositions which the modal syllogistic governs on Kilwardby's reading. As he emphasizes, this is a weaker condition than requiring all propositions to be *per se* necessities,⁸⁷ and is also independent from the requirement that they be necessities in the broad sense of inseparability. Being *secundum se* only requires that a proposition have a *per se* subject, and such propositions can be true merely as-of-now: For instance, 'all humans are walking'.⁸⁸ However, given Kilwardby's analysis of *per se* predication in terms of *suppositio* theory, it suffices for the necessity of a true assertoric proposition that its predicate be a *per se* term, and it suffices for the *per se* necessity of a true assertoric that its subject and predicate both be *per se* terms.⁸⁹ Hence, in Barbara LXL, if all propositions are read as having *per se* subject terms, the minor inherits a *per se* predicate from the major. It can therefore be upgraded to be a *per se* necessity. The premise pair of a Barbara LXL syllogism is therefore semantically equivalent to the corresponding pair of *per se* necessity propositions when all propositions are assumed to be *secundum se*.

This gives a more charitable way to read Kilwardby's claim that the minor of Barbara LXL is a necessity *secundum rem*. Kilwardby is making a claim about semantic equivalence. Barbara LXL does *not* require an explicitly modalized proposition to be substituted for the minor premise: The premise is an assertoric syntactically, or, in Kilwardby's terminology, *secundum vocem*.⁹⁰ However, given that the major is *secundum se*, the predicate of the minor is also a *per se* term. As a result, the minor can only be true under the same circumstances that would make a *per se* necessity true: Namely, that its subject falls under its predicate in the categorial order. Hence, an assertoric minor is semantically equivalent to a *per se* necessity in the presence of a *per se* necessity major. It is in this sense that the major 'appropriates' the minor to itself. That the minor is appropriated to a '*simpliciter*' proposition means that it is rendered semantically equivalent to a *per se* necessity.⁹¹ On the other hand, it can be shown that a true *per se* necessity which has as its predicate a term that is not *per se* does not semantically entail that the latter proposition is a necessity.⁹² For this reason, a *per se* necessity minor fails to 'appropriate' the major in the first figure.⁹³ Hence Barbara LXL is valid, but Barbara LLL is invalid.⁹⁴

⁸⁵ [...] *cum maior sit de necessario appropriat sibi minorem ita quod oportet ipsam esse de inesse simpliciter et minorem extremitatem accipi essentialiter sub medio, ita quod minor sit necessaria secundum rem. Maior autem cum sit de inesse (et minor non potest appropriare sibi maiorem, sed econverso), non oportet illam maiorem esse de inesse simpliciter sed bene poterit esse de inesse ut nunc.* Lectio 15:255–65 (dub. 7).

⁸⁶ See p. 11 of this paper.

⁸⁷ Cannone 2003–2004, p. 212:74.

⁸⁸ A similar observation is made by Thom 2007 (p. 21).

⁸⁹ As proven in the appendix. See lemma A.16.

⁹⁰ See Thom 2007 (p. 14).

⁹¹ This reading has the consequence that Kilwardby's claim that a necessity major appropriates the minor to be a *simpliciter* assertoric does not rule out the occurrence of *ut nunc* propositions in the mixed mood syllogistic. No semantic restrictions are placed on the kinds of assertoric propositions which can occur in the mixed mood necessity syllogistic, and an *ut nunc* major is compatible with a necessity minor in the first figure; nevertheless, it is a consequence of Kilwardby's semantics that the only sort of assertoric compatible with a necessity major in the first figure is one which is in fact necessary. For a different view, see Thom 2007 (p. 243).

⁹² See theorem A.14.

⁹³ Lectio 15:258 (dub.7).

⁹⁴ This is shown to follow from Kilwardby's semantics in appendix section A.6.

4. Concluding remarks

Kilwardby's solution to necessity conversion and the Two Barbaras can be viewed as a 'term restriction', but in a different sense to what we have seen in Rini 2011. Kilwardby effectively introduces, alongside each standard propositional form, a modified form which is true just when the original proposition is true and the subject is a *per se* term. He then views Aristotle's necessity syllogistic as a logic for such modified propositional forms. This renders key results in the modal syllogistic formally valid in the schematic sense.⁹⁵ For example, under the semantics we have reconstructed from Kilwardby's remarks, the schema:

$$\frac{Aa_{L\sigma}B}{\frac{Ba_{\sigma}C}{Aa_{L\sigma}C}} \quad (27)$$

is valid subject to the *non*-restrictive side-condition:

$$\begin{aligned} \mathfrak{F} &= \{L, \sigma, a, e, i, o\} \\ \mathfrak{V} &= \{A, B, C\} \\ \text{Ran}(A) &= \text{Ran}(B) = \text{Ran}(C) = \{\text{Terms in the language}\}. \end{aligned} \quad (28)$$

Hence, if we require all propositions to be *secundum se*, Barbara LXL becomes formally valid in the sense defined on p. 7. Similar remarks hold for the other first-figure moods. Since Aristotle proves the validity of syllogisms in the second and third figures by reducing them to first figure syllogisms by means of conversion, these results are central to Aristotle's proofs in the mixed necessity-assertoric syllogistic. It is nevertheless not straightforward to extend these results to the remainder of the modal syllogistic, and it falls beyond the scope of this paper to consider to what extent a term-kind-based interpretation is ultimately sustainable.⁹⁶

Even if there are elements of Kilwardby's commentary which in the end 'militate against' (Thom 2007, p. 5) interpreting him as putting forward a formal system of syllogistic in the modern sense, however, we can conclude that conversion and the problem of the Two Barbaras are, contrary to Thom, not among these. Kilwardby's way of dealing with the problem of the Two Barbaras and modal conversion in fact shows that the connection between the formal validity of Aristotle's modal syllogistic and its reliance on term kinds is more subtle than modern commentators have taken it to be. We need not assume, as Rini does, that any logic which relies on a distinction between kinds of terms must for that reason rely on a non-standard notion of validity.

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⁹⁵ Note that this is consistent with Kilwardby using the term 'form' as applied to syllogisms to mean something other than logical form in the modern sense. Cf. Thom 2013. However, see also Brumberg-Chaumont 2015 (p. 264), who argues against Thom that Kilwardby sides with the Anonymous Aurelianensis II and III against the Anonymous Cantabrigiensis in defending a 'purely syntactic conception of syllogistic form'.

⁹⁶ Kilwardby's interpretation of negatives and contingency propositions is complex, and it is not clear that he takes these to be reducible to the notions of *per se* in the first sense and third senses. Even within the affirmative syllogistic, the interpretation presented here will not validate Darapti LXL, Darapti XNN, Disamis XNN and Datisi NXN as it stands. These moods have in common that they require the use of *assertoric* conversion. While requiring subject terms to be *per se* ensures the convertibility of necessities, it in fact invalidates conversion for assertoric propositions, since the converted proposition is not guaranteed a *per se* subject. Kilwardby does not show any awareness of this problem when he treats these moods in Lectio 17. This can be viewed as a limitation of the term-kind approach, or at least of Kilwardby's implementation of it.

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A. Appendix. Semantics for syllogistic necessities

A semantics is given for assertoric and necessity syllogistic logic based on Kilwardby's interpretation. Rigorous proofs are given of the main results discussed above using the semantic entailment relation generated by this semantics.

A.1. Language

A term-logical language \mathcal{L} consists of the following symbols:

A collection of terms $\mathcal{T} = \{T_1, T_2, \dots\}$

Four atomic copulas, with the following intended readings:

a – 'is said of all'

i – 'is said of some'

e – 'is said of no'

o – 'is not said of all'

Three copula modifiers:

L – necessarily

\mathbf{L} – *per se* necessarily

σ – *secundum se*

If \star is an atomic copula and \dagger, \ddagger are distinct copula modifiers, then $\star\dagger$ and $\star\dagger\ddagger$ are copulas. If A, B are terms and \circ is a copula, then $A \circ B$ is a well-formed formula. Nothing else is a well-formed formula.

A.2. Models

Definition A.1 A Kilwardby model for a language \mathcal{L} with terms \mathcal{T} is a structure $\langle I, F, \leq, Con, Sup, Form \rangle$, where:

I is a non-empty set.

F is a non-empty set.

\leq is a transitive, reflexive, anti-symmetric relation over F .

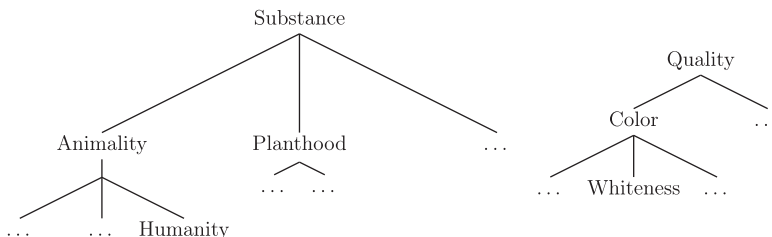


Figure A1. Forms are ordered by \leq into a genus-species tree. Neither binary branching nor a unique root is required.

$$\begin{aligned} \text{Con}: T &\rightarrow F \\ \text{Sup}: T &\rightarrow \mathcal{P}(I) \\ \text{Form}: I &\rightarrow F \end{aligned}$$

In addition to being a partial ordering, \leq is required to satisfy the following conditions:

Condition A.2 (Downward branching) For any X, Y, Z ,
if $X \leq Y$ and $X \leq Z$, then $Y \leq Z$ or $Z \leq Y$

Condition A.3 (No abstract supposita) For all $x \in I$, there is no $f \in F$ such that $f \leq \text{Form}(x)$ and $\text{Form}(x) \not\leq f$.

Con, *Sup* and *Form* are connected by the following condition:

Condition A.4 For any term T and $x \in I$,
if $\text{Form}(x) \leq \text{Con}(T)$ then $x \in \text{Sup}(T)$.

To define the final semantic conditions on a model, we need to define the important concept of *per se* term.

Definition A.5 T is *per se* if, and only if, $\forall x \in \text{Sup}(T): \text{Form}(x) \leq \text{Con}(T)$

This definition is illustrated in Figure A2. We write $\sigma_{\mathfrak{M}}T$ to mean that T is *per se* in model \mathfrak{M} . When \mathfrak{M} is clear from the context, the subscript is omitted. We can now state the final condition which a structure must satisfy to count as a model.

Condition A.6 For terms T, S , if $\text{Con}(T) \neq \text{Con}(S)$ and $\text{Con}(T) \leq \text{Con}(S)$, then:

- (1) σT if, and only if, σS
- (2) $\text{Sup}(T) \subset \text{Sup}(S)$

A model assigns two interpretations to each term: *Con* ('cosignification') and *Sup* ('supposition'). I can be thought of as a set of individuals, and $\text{Sup}(T)$ can be thought of as 'the Ts' (a subset of I) for each term T . In addition to a domain of individuals, models include a domain of forms F . $\text{Con}(T)$, a member of F , can be thought of as 'being T', 'T-ness' or 'T-ity'. For example $\text{Sup}(\text{Human})$ is the set of all humans, while $\text{Con}(\text{Human})$ is the form associated with the term 'human', *humanity*. Both terms and individuals are assigned forms in a model, by *Con* and *Form* respectively. Models do not stipulate whether individuals which are suppositated by a term T (the members of the set $\text{Sup}(T)$) have the form which is cosignified by T ($\text{Con}(T)$). If they all do, then the term is said to be *per se*; if they do not, then the term is said to be *per accidens* (see Figure A2).

The domain of forms is ordered by \leq . Conditions are placed on \leq so as to capture the tree structure which Porphyry and Boethius refer to.⁹⁷ Condition A.3 encodes the assumption that the individuals belonging to a term's supposita are 'concrete': Completive forms must not be superior to any other forms on the genus-species tree.

Condition A.4 is an immediate consequence of the intuitive meaning of *Sup* and *Con*. It states that if an individual's form falls under the form cosignified by a term T , then that individual is suppositated for by T . For instance, if the form of Socrates is humanity, and humanity falls under animality, then Socrates is required to be one of the animals. The first clause of Condition A.6 states that if a term is *per se*, then all of the terms on its branch

⁹⁷ See footnote 41. Compare the 'preorder semantics' of Malink 2013 (pp. 73–85).

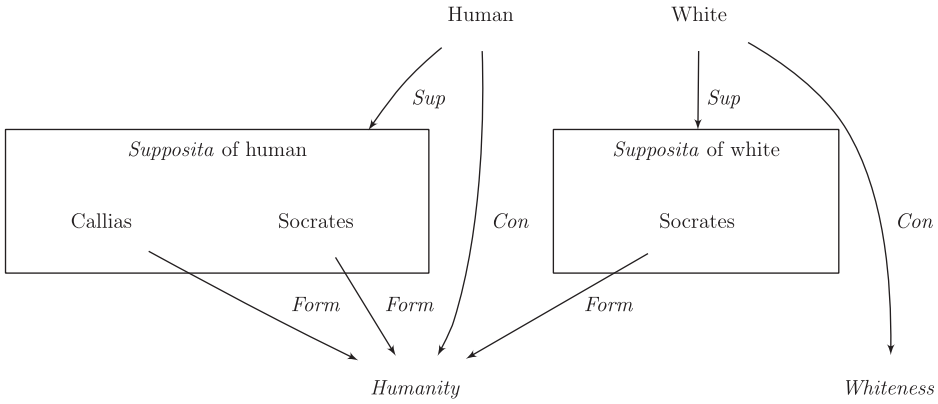


Figure A2. In this model, ‘human’ is a *per se* term, since all of its *supposita* (Callias, Socrates) have the form *humanity*, which is also the form cosignified by the term ‘human’. Assuming $\text{Humanity} \not\subseteq \text{Whiteness}$ (as in the tree above), ‘white’ will not be a *per se* term, since something which is white (Socrates, in this example) has a form (*humanity*) which does not fall under the cosignification of white (*whiteness*).

are *per se* as well.⁹⁸ The second clause is a consequence of the intuitive interpretation of *Sup* and *Con*. It rules out the possibility that the form of X is subordinated to the form of Y but the Xs are not among the Ys. The set inclusion is strict, encoding the assumption that division into kinds must genuinely narrow down the individuals in question: We exclude models where X is a kind of Y, but that there are no more Ys than Xs.

A.3. Truth and consequence

For a given model \mathfrak{M} , we write $\mathfrak{M} \models A$ to mean A is true in \mathfrak{M} . For any terms A and B, the conditions for the truth of propositions containing unmodified and necessity copulae are as follows.

Definition A.7 Truth conditions

- $\mathfrak{M} \models AaB$ if, and only if, $\text{Sup}(B) \neq \emptyset$ and $\text{Sup}(B) \subseteq \text{Sup}(A)$
- $\mathfrak{M} \models AiB$ if, and only if, $\text{Sup}(B) \cap \text{Sup}(A) \neq \emptyset$
- $\mathfrak{M} \models AeB$ if, and only if, $\text{Sup}(B) \cap \text{Sup}(A) = \emptyset$
- $\mathfrak{M} \models AoB$ if, and only if, $\text{Sup}(B) \not\subseteq \text{Sup}(A)$
- $\mathfrak{M} \models Aa_LB$ if, and only if, $\text{Sup}(B) \neq \emptyset$ and $\forall x \in \text{Sup}(B): \text{Form}(x) \leq \text{Con}(A)$
- $\mathfrak{M} \models Ai_LB$ if, and only if, $\exists x \in \text{Sup}(B): \text{Form}(x) \leq \text{Con}(A)$
- $\mathfrak{M} \models Ae_LB$ if, and only if, $\neg \exists x \in \text{Sup}(B): \text{Form}(x) \leq \text{Con}(A)$
- $\mathfrak{M} \models Ao_LB$ if, and only if, $\neg \forall x \in \text{Sup}(B): \text{Form}(x) \leq \text{Con}(A)$
- $\mathfrak{M} \models Aa_LB$ if, and only if, $\sigma A, \sigma B$ and $\text{Con}(B) \leq \text{Con}(A)$
- $\mathfrak{M} \models Ai_LB$ if, and only if, $\sigma A, \sigma B$ and $\text{Con}(B) \leq \text{Con}(A)$ or $\text{Con}(A) \leq \text{Con}(B)$
- $\mathfrak{M} \models Ae_LB$ if, and only if, $\sigma A, \sigma B$ and $\text{Con}(B) \not\leq \text{Con}(A)$ and $\text{Con}(A) \not\leq \text{Con}(B)$
- $\mathfrak{M} \models Ao_LB$ if, and only if, $\sigma A, \sigma B$ and $\text{Con}(B) \not\leq \text{Con}(A)$

Assertorics are interpreted extensionally but, as we will see, they become equivalent to necessities of the two respective types when their predicate and subject terms are respectively *per se* (Section A.5). A necessity, in the weak sense, expresses that some or all individuals falling under the subject term have or fail to have the form signified by the predicate. This is intended to reflect Kilwardby’s claim that the predicate has simple supposition, while the subject has personal supposition, in necessities interpreted broadly (cf. p. 16 of this paper). *Per se* necessities are defined in terms of *per se* inherence (cf. p. 10–2).

⁹⁸ See Malink 2013 (p. 146) for a rationale for this requirement in Aristotle.

We now give the truth conditions for propositions involving copulas modified by σ ('*secundum se*'). This operator modifies the truth condition of a propositional form so as to require the subject term to be *per se*.

Definition A.8 For any model \mathfrak{M} , if \circ is a copula, then

$$\mathfrak{M} \models A \circ_{\sigma} B \quad \text{if, and only if, } \sigma_{\mathfrak{M}} B \text{ and } \mathfrak{M} \models A \circ B$$

To account for the remaining formulae in the language, we stipulate that $\mathfrak{M} \models A \circ_{\sigma L} B$ if, and only if, $\mathfrak{M} \models A \circ_{L\sigma} B$ for any $\mathfrak{M}, A, B, \circ$ (that is, the order of modifiers does not affect truth; a *secundum se* necessity is true under the same conditions as a necessity *secundum se*).

We define semantic entailment in the usual way.

Definition A.9 For a well-formed formula $Q \in \mathcal{L}$ and a list of well-formed formulae $P_1 \in \mathcal{L}, P_2 \in \mathcal{L}, \dots, P_n \in \mathcal{L}$, we write $P_1, P_2, \dots, P_n \models Q$ to mean that for all Kilwardby models \mathfrak{M} , if $\mathfrak{M} \models P_1$ and $\mathfrak{M} \models P_2$ and \dots and $\mathfrak{M} \models P_n$, then $\mathfrak{M} \models Q$.

A.4. Conversion

It is clear from inspection that *per se* necessities and assertorics satisfy conversion. On the other hand, necessities in the weak sense fail to convert:

THEOREM A.10 *Conversion fails for necessities in the weak sense.*

$$(i) Aa_L B \not\models Bi_L A \quad (ii) Ai_L B \not\models Bi_L A, \\ (iii) Ae_L B \not\models Be_L A.$$

Proof We show this for the affirmatives only. Take $\mathfrak{M} = \langle I, F, \leq, Con, Sup, Form, \rangle$ with $I = \{a\}, F = \{f, g\}, \leq = \{\langle f, f \rangle, \langle g, g \rangle\}, Con(A) = f, Con(B) = g, Sup(A) = Sup(B) = \{a\}, Form(a) = f$. Then $\mathfrak{M} \models Aa_L B$ and $\mathfrak{M} \models Ai_L B$, but $\mathfrak{M} \not\models Bi_L A$. \blacksquare

This result captures Kilwardby's observation that conversion fails for necessity interpreted broadly to include necessities *per accidens*. Such necessities do however convert when they are required to have *per se* subjects and non-empty terms, since in that case they become semantically equivalent to *per se* necessities, as we will now show.

A.5. *Secundum se* propositions and *per se* necessities

We first prove the following preliminary result. Unlike *per se* necessities, affirmative necessities in the weak sense do imply the corresponding assertorics:

LEMMA A.11 *Affirmative necessities imply the corresponding assertorics.*

$$(i) Aa_L B \models AaB \quad (ii) Ai_L B \models AiB.$$

Proof (i): Suppose $\mathfrak{M} \models Aa_L B$. Then $\forall x \in Sup(B) : Form(x) \leq Con(A)$ in \mathfrak{M} by the definitions in Section A.7. By Condition A.4, $Form(s) \leq Con(A)$ implies $s \in Sup(A)$. Substituting yields $\forall x \in Sup(B) : x \in Sup(A)$, that is, $Sup(B) \subseteq Sup(A)$. Hence $\mathfrak{M} \models AaB$. \mathfrak{M} was arbitrary, so $Aa_L B \models AaB$. The proof of (ii) is analogous. \blacksquare

We now prove that necessities with non-empty *per se* terms are equivalent to *per se* necessities:

THEOREM A.12 *If $Sup(A) \neq \emptyset$ and $Sup(B) \neq \emptyset$ in \mathfrak{M} , then $\mathfrak{M} \models A \circ_{L\sigma} B$ if, and only if, $\mathfrak{M} \models A \circ_L B$ (for $\circ = a, i, e, o$).*

Proof We show here only the affirmative results.

\Rightarrow : Suppose $\mathfrak{M} \models A \circ_{L\sigma} B$. Then B is *per se*, so

$$\forall x \in Sup(B) : Form(x) \leq Con(B). \quad (A1)$$

If $\circ = i$, then $\mathfrak{M} \models Ai_L B$, so

$$\exists x \in Sup(B) : Form(x) \leq Con(A). \quad (A2)$$

Instantiating (A2) and calling the object b , we get $Form(b) \leq Con(A)$ and so, by (A1), $Form(b) \leq Con(B)$. By Condition A.2, then,

$$Con(B) \leq Con(A) \text{ or } Con(A) \leq Con(B). \quad (A3)$$

Furthermore, A is *per se* by Condition A.6. Hence

$$\sigma A, \sigma B \text{ and } (Con(B) \leq Con(A) \text{ or } Con(A) \leq Con(B)), \quad (A4)$$

which is the truth condition for a *per se* particular necessity.

If $\circ = a$, then $\mathfrak{M} \models Aa_{L\sigma}B$, so

$$\forall x \in \text{Sup}(B) : \text{Form}(x) \leq \text{Con}(A). \quad (\text{A5})$$

Since $\text{Sup}(B)$ is non-empty, we can take $b \in \text{Sup}(B)$. By (A1) and (A5) we again get $\text{Form}(b) \leq \text{Con}(B)$ and $\text{Form}(b) \leq \text{Con}(A)$, and by Condition A.2, (A4) holds.

If $\text{Con}(A) = \text{Con}(B)$, then $\text{Con}(B) \leq \text{Con}(A)$ by reflexivity. Suppose now for *reductio* that $\text{Con}(A) \leq \text{Con}(B)$ and $\text{Con}(A) \neq \text{Con}(B)$. Then $\text{Sup}(A) \subset \text{Sup}(B)$ by Condition A.6. But, by definition A.7, $\mathfrak{M} \models AaB$. So $\text{Sup}(B) \subseteq \text{Sup}(A)$. By *reductio*, it follows that, $\text{Con}(A) \not\leq \text{Con}(B)$. Hence, by (A3)

$$\text{Con}(B) \leq \text{Con}(A). \quad (\text{A6})$$

Hence we can eliminate the right disjunct of (A4) to obtain:

$$\sigma A, \sigma B \text{ and } \text{Con}(B) \leq \text{Con}(A), \quad (\text{A7})$$

which is the truth condition for a universal affirmative *per se* necessity.

←: Suppose $\text{Con}(B) \leq \text{Con}(A)$ in \mathfrak{M} and B is *per se*. If $\text{Con}(B) = \text{Con}(A)$, then $\text{Sup}(B) \subseteq \text{Sup}(A)$. Otherwise, by Condition A.6,

$$\text{Sup}(B) \subseteq \text{Sup}(A). \quad (\text{A8})$$

Again by Condition A.6, A is *per se*, so

$$\forall x \in \text{Sup}(A) : \text{Form}(x) \leq \text{Con}(A) \quad (\text{A9})$$

(A8) and (A9) give:

$$\forall x \in \text{Sup}(B) : \text{Form}(x) \leq \text{Con}(A). \quad (\text{A10})$$

Hence $\mathfrak{M} \models Aa_{L\sigma}B$. But B is *per se*, so $\mathfrak{M} \models Aa_{L\sigma}B$. Hence if $\mathfrak{M} \models Aa_{L\sigma}B$, $\mathfrak{M} \models Aa_{L\sigma}B$. Since $\text{Sup}(B)$ is non-empty, (A10) implies:

$$\exists x \in \text{Sup}(B) : \text{Form}(x) \leq \text{Con}(A) \quad (\text{A11})$$

and so if $\text{Con}(B) \leq \text{Con}(A)$, $\mathfrak{M} \models Ai_{L\sigma}B$.

Suppose $\text{Con}(A) \leq \text{Con}(B)$ in \mathfrak{M} . Then, since A is *per se*, by the transitivity of \leq ,

$$\forall x \in \text{Sup}(A) : \text{Form}(x) \leq \text{Con}(B) \quad (\text{A12})$$

but by Condition A.6, $\text{Sup}(A) \subseteq \text{Sup}(B)$, so, since $\text{Sup}(B)$ is non-empty

$$\exists x \in \text{Sup}(B) : \text{Form}(x) \leq \text{Con}(A). \quad (\text{A13})$$

And so if $\text{Con}(A) \leq \text{Con}(B)$ then $\mathfrak{M} \models Ai_{L\sigma}B$.

Hence if $\sigma A, \sigma B$ and $\text{Con}(A) \leq \text{Con}(B)$ or $\text{Con}(B) \leq \text{Con}(A)$, (that is, if $\mathfrak{M} \models Ai_{L\sigma}B$), $\mathfrak{M} \models Ai_{L\sigma}B$. ■

Since **L**-conversion rules are valid, it follows immediately from this that $L\sigma$ -conversion rules are valid when terms are non-empty:

COROLLARY A.13 *If $\text{Sup}(A) \neq \emptyset$ and $\text{Sup}(B) \neq \emptyset$ in \mathfrak{M} then*

$$(i) \text{ If } \mathfrak{M} \models Aa_{L\sigma}B, \mathfrak{M} \models Bi_{L\sigma}A \quad (ii) \text{ If } \mathfrak{M} \models Ai_{L\sigma}B, \mathfrak{M} \models Bi_{L\sigma}A$$

$$(iii) \text{ If } \mathfrak{M} \models Ae_{L\sigma}B, \mathfrak{M} \models Be_{L\sigma}A.$$

A.6. Syllogistic results

We show that the desired syllogistic results in the first figure hold for *secundum se* propositions. The pure assertorics are trivial (the minor premise guarantees that C is a *per se* term). The modal results are proven for Barbara. The proofs require only minimal modification to yield an analogue of each result for other first-figure moods.

THEOREM A.14 *Invalidity of Barbara $X_{\sigma}L_{\sigma}L_{\sigma}$: $Aa_{\sigma}B, Ba_{L\sigma}C \not\models Aa_{L\sigma}C$*

Proof Take $\mathfrak{M} = \langle I, F, \leq, \text{Con}, \text{Sup}, \text{Form} \rangle$ with:

$$I = \{i\} \quad F = \{a, b, c\}$$

$$\text{Con}(A) = a \quad \text{Sup}(A) = \{i\}$$

$$\text{Con}(B) = b \quad \text{Sup}(B) = \{i\}$$

$$\text{Con}(C) = c \quad \text{Sup}(C) = \{i\}$$

$$\text{Form}(i) = c$$

$$\leq = \{\langle c, b \rangle, \langle c, c \rangle, \langle b, b \rangle, \langle a, a \rangle\}.$$

It can be verified that \mathfrak{M} is a model. Further, $\mathfrak{M} \models Aa_{\sigma}B$ and $\mathfrak{M} \models Ba_{L\sigma}C$, but $\mathfrak{M} \not\models Aa_{L\sigma}C$. ■

THEOREM A.15 *Barbara* $L_\sigma L_\sigma L_\sigma : Aa_{L_\sigma} B, Ba_{L_\sigma} C \models Aa_{L_\sigma} C$.

Proof Since $Sup(C)$ is non-empty, all terms must have non-empty supposition (lemma A.11). The result then follows from theorem A.12 and the transitivity of \leq . ■

In order to prove Barbara $L_\sigma X_\sigma L_\sigma$, we first establish a lemma:

LEMMA A.16 *Upgrading*: If $\sigma_{\mathfrak{M}} A$ and $\mathfrak{M} \models A \circ_\sigma B$, then $\mathfrak{M} \models A \circ_{L_\sigma} B$.

Proof We prove the case for the a copula. Suppose $\mathfrak{M} \models AaB$ and A is *per se* in \mathfrak{M} . Then we can reason about \mathfrak{M} as follows:

- | | | |
|---|--|--|
| 1 | $Sup(B) \subseteq Sup(A)$ | since $\mathfrak{M} \models Aa_\sigma B$ |
| 2 | σB | since $\mathfrak{M} \models Aa_\sigma B$ |
| 3 | $Sup(B) \neq \emptyset$ | since $\mathfrak{M} \models Aa_\sigma B$ |
| 4 | $\forall x \in Sup(A) : Form(x) \leq Con(A)$ | since $\sigma_{\mathfrak{M}} A$ |
| 5 | $\forall x \in Sup(B) : Form(x) \leq Con(A)$ | by set theory from 1 and 4 |
| 6 | $\mathfrak{M} \models Aa_{L_\sigma} B$ | by 2, 3 and 5 |

THEOREM A.17 *Barbara* $L_\sigma X_\sigma L_\sigma : Aa_{L_\sigma} B, Ba_\sigma C \models Aa_{L_\sigma} C$.

Proof By the definition of \circ_σ , B is *per se*. Hence we can use lemma A.16 to upgrade $Ba_\sigma C$ to $Ba_{L_\sigma} C$. The premise pair is therefore equivalent to $Aa_{L_\sigma} B, Ba_{L_\sigma} C$. The conclusion $Aa_{L_\sigma} C$ then follows from Theorem A.15. ■